Teorie automatického řízení
Studijní opory a návody

Theory of Automatic Control – Study supports and instructions

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Bakalářská práce
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Univerzita Tomáše Bati ve Zlíně
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Univerzita Tomáše Bati ve Zlíně
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Ústav automatizace a řídicí techniky
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ZADÁNÍ BAKALÁŘSKÉ PRÁCE

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Jméno a příjmení:	**Pavel ELŠÍK**
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Téma práce:	**Teorie automatického řízení -- Studijní opory a návody**

Zásady pro vypracování:

Práce se bude zabývat vytvořením studijních materiálů pro účely předmětu TAŘ-1 v anglickém jazyce. Výsledkem budou ppt prezentace, www materiály, texty, vzorové příklady a protokoly z uvedené oblasti. Vhodné a kvalifikované prostředí pro simulaci a výpočty je MATLAB, Simulink. V práci půjde zejména o následující úkoly:

1. **Příprava stránek ppt z přednášek předmětu.**
2. **Vizualizace schémat a pojmů v teorii automatického řízení.**
3. **Tvorba www stránek předmětu.**
4. **Příklady charakteristik lineárních systémů a simulací (Matlab, Simulink).**
5. **Vytvoření vzorových protokolů.**

Rozsah práce:

Rozsah příloh:

Forma zpracování bakalářské práce: **tištěná/elektronická**

Seznam odborné literatury:

1. **Prokop, R. a kol. Teorie automatického řízení. Skriptum FAI UTB, Zlín 2006.**

2. **Balátě, J.: Teorie řízení. BEN, Praha 1982.**

3. **Šulc, B., Vítečková, M.: Teorie a praxe návrhu regulačních obvodů. ČVUT Praha, 2004.**

4. **Vítečková, M.: Víteček, A.: Anglicko-český slovník pojmů AŘ. VŠB Ostrava, 2006.**

5. **Zaplatílek, K., Doňar, B.: MATLAB pro začátečníky. BEN Praha 2005.**

6. **Bishop, R.H.: Modern control systems using Matlab and Simulink. Adison Wesley, Menlo Park, 1997.**

7. **Kuo, B.C.: Automatic control systems. Prentice Hall, Englewood Cliffs, 1995.**

Vedoucí bakalářské práce: **prof. Ing. Roman Prokop, CSc.**
Ústav automatizace a řídicí techniky

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Ve Zlíně dne 22. února 2008

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prof. Ing. Vladimír Vašek, CSc.
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ředitel ústavu

ABSTRAKT

Cílem této práce je vytvoření studijních návodů a opor pro předmět Teorie automatického řízení I. V teoretické části je stručně nastíněn obsah prezentací tvořených v programu PowerPoint. V praktické části jsou samotné prezentace a také vzorové protokoly. Prezentace i protokoly jsou vypracovány v anglickém jazyce a měli by sloužit k výuce zahraničních studentů studujících na naší fakultě. Simulace pochodů ve vzorových protokolech jsou provedeny v prostředí MATLAB/SIMULINK.

Klíčová slova:

Lineární spojité dynamické systémy, laplaceova transformace, přenos systému, stabilita, regulační obvod, regulátor

ABSTRACT

The aim of this work is creating study instructions and supports for the subject Theory of automatic control I which is held in the bachelor study. The theoretical part outlines the brief content of further subject presentations in PowerPoint environment. In practical part then the slides of presentations follow as well as sample exemplary laboratory protocols. Both of them are in English language and intended for foreign students studying in the faculty. Simulations of behaviour in exemplary protocols are performed in the program MATLAB/SIMULINK.

Keywords:

Linear continuous dynamic systems, laplace transform, transfer function, stability, control system, controller.

Rád bych touto cestou poděkoval prof. Ing. Romanu Prokopovi, Csc. za vedení bakalářské práce, za poskytování odborných rad a za zapůjčení literatury z oblasti automatizace.

Prohlašuji, že jsem na bakalářské práci pracoval samostatně a použitou literaturu jsem citoval. V případě publikace výsledků, je-li to uvolněno na základě licenční smlouvy, budu uveden jako spoluautor.

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Podpis diplomanta

OBSAH

ÚVOD

Proces automatizace proniká téměř do všech oblastí společenského života od materiální výroby přes organizaci, plánování, až po řízení společenských procesů. Automatizace výrobních procesů přináší: zjednodušení, zkrácení doby výroby, zvýšení kvality, zefektivnění práce, snížení výrobních nákladů, zvýšení stability výrobního procesu,aj. Automatizace umožňuje přesné a rychlé změření, vyhodnocení naměřených hodnot a provedení potřebného zásahu. Také umožňuje zavedení rozsáhlé operační a mezioperační kontroly bez zvýšení počtu kontrolních pracovníků. Odstranění lidských faktorů z výrobního procesu zvyšuje jeho kvalitu i spolehlivost a zvyšuje jeho přesnost. Člověk se zbavuje těžké, fyzicky náročné práce a uvolňuje se pro složitější a náročnější tvůrčí činnost. Bez automatizace se neobejde žádný výrobní nebo technologický proces.

Úkolem předmětu je nastínit studentům problematiku automatického řízení a seznámit je se základními pojmy z této oblasti. Tato bakalářská práce je zaměřena na vytvoření studijních návodů a opor pro předmět Teorie automatického řízení I. Jednak byly pomocí programu PowerPoint vytvořeny prezentace, které budou sloužit k podpoře přednášek a jednak také vzorové protokoly ve formě pdf sloužící na podporu laboratorních cvičení, to vše v anglickém jazyce.

I. TEORETICKÁ ČÁST

1 ÚVOD DO TEORIE SYSTÉMŮ

Systém – je soubor prvků, mezi nimiž existují vzájemné vztahy a jako celek má určité vztahy ke svému okolí.

Každý systém je charakterizován dvěma základními vlastnostmi:

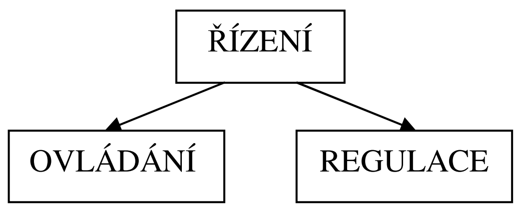
1. chování systému, charakterizujícím jeho vnější vztahy k okolí. Chování systému je závislost mezi podněty okolí systému působícími na jeho vstup a příslušnými odezvami objevujícími se na jeho výstupu.

2. strukturou systému, charakterizující jeho vnitřní funkční vztahy. Strukturou systému rozumíme jednak způsob uspořádání vzájemných vazeb mezi prvky systému a jednak chování těchto prvků.

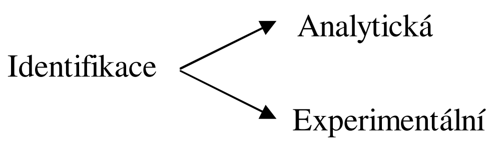
Obě tyto vlastnosti systému jsou ve velmi úzkém vztahu, který lze charakterizovat jednak, že určité struktuře odpovídá jednoznačně určité chování a naopak, že určitému chování odpovídá třída struktur, definovaná tímto chováním.

Řízení – je cílevědomé působení na objekt s cílem zajistit žádané chování tohoto objektu.

Řízení můžeme rozdělit na ovládání (otevřené řízení, řízení bez zpětné vazby) a regulaci (uzavřené řízení, zpětnovazební řízení)



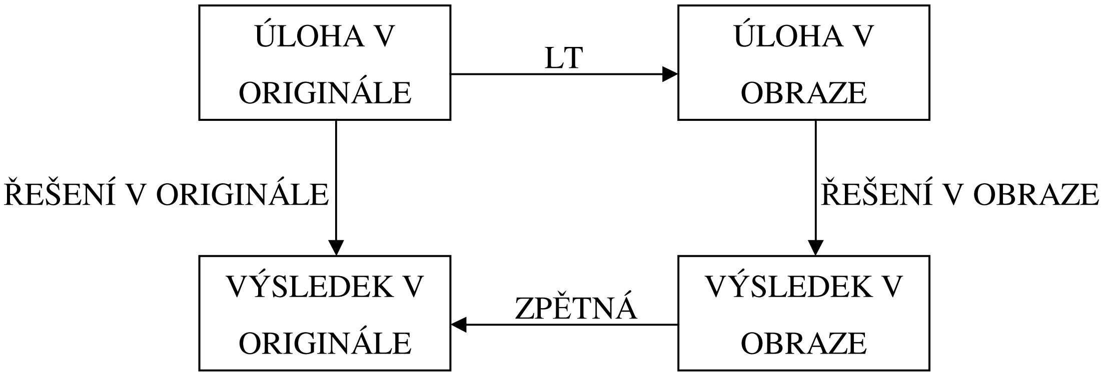
Pro teorii automatického řízení má velký význam redukce systému na jeho matematický model, která se nazývá identifikace.



2 LAPLACEOVA TRANSFORMACE

Laplaceova transformace (L-transformace) představuje velmi účinný nástroj při popisu chování tj. analýze a syntéze, spojitých dynamických systémů.

Účelem transformace je převést složitou úlohu z prostoru originálu do prostorů obrazů, kde se tato úloha vyřeší velmi snadno a pak se převede zpět do prostoru originálu.



Obr. 1. Postup výpočtu při použití Laplaceovy transformace

2.1 Přímá Laplaceova transformace

(Určení obrazu k danému originálu)

- pomocí vztahu

$$F(s) = L\{f(t)\} = \int_0^\infty f(t) e^{-st} dt$$

kde

$f(t)$ … originál – reálná funkce definovaná v časové oblasti $t \in \langle 0, \infty)$

$F(s)$ … obraz – komplexní funkce definovaná v oblasti komplexní proměnné

Aby funkce $f(t)$ byla originálem, musí být:

1. nulová pro záporný čas, tj.

$$f(t) = \begin{cases} f(t) & \text{pro} \quad t \geq 0, \\ 0 & \text{pro} \quad t < 0, \end{cases}$$

2. alespoň po částech spojitá,

3. exponenciálního řádu, tj. musí vyhovovat nerovnosti

$$\left|f(t)\right| \le M e^{\alpha_0 t},$$

kde M > 0; α0 ∈ (-∞, ∞), t ∈ ⟨0, ∞)

2.2 Zpětná Laplaceova transformace

- pomocí vztahu

$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j}\oint F(s)e^{st}ds \rightarrow f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{\alpha_0 - j\varpi}^{\alpha_0 + j\varpi} F(s)e^{st}ds$$

- pomocí věty o residuích

pro originál *f*(t) platí:

$$f(t) = \sum_i \mathop{res}_{s=p_i}\left[F(s)e^{st}\right]$$

kde $s = p_i$ jsou póly funkce *F*(s) a $res\left[F(s)e^{st}\right]$ jsou residua pro jednotlivé póly p_i.

Pro *n*-násobný pól platí:

$$res\left[F(s)e^{st}\right] = \frac{1}{(n-1)!}\lim_{s\to p_i}\frac{d^{n-1}}{ds^{n-1}}\left[(s-p_i)^n F(s)e^{st}\right]$$

n je násobnost (řád) singulárního bodu (pólu) obrazu *F*(s)

Pro nenásobný pól (*n* = 1) platí:

$$res\left[F(s)e^{st}\right] = \lim_{s\to p_i}\left[(s-p_i)F(s)e^{st}\right]$$

- pomocí slovníku

Pro jednodušší funkci *F*(s) lze použít (k získání originálu *f*(t)) přímo slovník.

Pro složitější funkci *F*(s) musíme nejprve tuto funkci rozložit na parciální zlomky a k těm poté najít originál *f*(t)ve slovníku. Rozklad na parciální zlomky lze provést jednak *metodou neurčitých koeficientů*, ale také použitím *Heavisideova rozvoje*.

3 LINEÁRNÍ SPOJITÉ DYNAMICKÉ SYSTÉMY (LSDS)

Chování spojitého systému s jednou vstupní a jednou výstupní veličinou lze popsat lineární diferenciální rovnicí s konstantními součiniteli ve tvaru:

$$y^{(n)}(t) + a_{(n-1)}y^{(n-1)}(t) + \ldots + a_1 y'(t) + a_0 y(t) = b_m u^{(m)}(t) + \ldots + b_0 u(t)$$

kde

a_i, b_j jsou konstantní koeficienty

u(t) – vstupní veličina

y(t) – výstupní veličina

Popis dynamických vlastností lineárního systému, lze rozdělit na dvě skupiny:

Vnější popis systému - vyjadřuje dynamické vlastnosti dějů mezi vstupem a výstupem systému. Při vnějším popisu systému je systém považován za černou skříňku se vstupem a výstupem. Analyzuje se pouze reakce systému na vstupní signály.

- lineární diferenciální rovnice systému,
- přenos systému (v Laplaceově transformaci),
- nuly a póly přenosu systému,
- přechodová funkce a charakteristika,
- impulsní funkce a charakteristika,
- frekvenční přenos,
- amplitudově-fázová frekvenční charakteristika v komplexní rovině (Nyquistova křivka),
- frekvenční charakteristiku v logaritmických souřadnicích (Bodého křivka).

Vnitřní popis systému –Vyjadřuje dynamické vlastnosti reakcí mezi vstupem, vnitřním stavem a výstupem systému. Vnitřní popis vede na tzv. *stavový model systému*.

Přenos systému - je definován jako poměr Laplaceova obrazu výstupní veličiny k Laplaceově obrazu vstupní veličiny při nulových počátečních podmínkách.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + \ldots + b_0}{s^n + a_{n-1}s^{(n-1)} + \ldots + a_1 s + a_0}$$

Nuly a póly přenosu systému

Póly jsou kořeny jmenovatele přenosu systému. Nuly jsou kořeny čitatele přenosu. Nuly a póly mohou být reálné, komplexně sdružené nebo ryze imaginární. Nuly rozhodují o fázovosti systému, póly rozhodují o stabilitě. Póly v počátku představují integrační charakter přechodového děje systému. Nuly v počátku určují derivační charakter přechodového děje systému. Póly a nuly ležící nalevo od imaginární osy jsou stabilní, kdežto ty co leží vpravo jsou nestabilní. Póly či nuly ležící na imaginární ose jsou tzv. na hranici stability.

Přechodová funkce a charakteristika

Přechodová funkce je odezva systému na jednotkový (Heavisideův) skok při nulových počátečních podmínkách. Přechodová charakteristika je grafické znázornění přechodové funkce. Tato funkce je označována *h*(t).

Impulsní funkce a charakteristika

Impulsní funkce je odezva systému na jednotkový (Diracův) impulz při nulových počátečních podmínkách. Impulsní charakteristika je grafické znázornění impulsní funkce. Tato funkce je označována *i*(t).

Frekvenční přenos

Frekvenční přenos je poměr Fouriérových obrazů vstupního a harmonického signálu ku výstupnímu při nulových počátečních podmínkách. Frekvenční přenos je značen $G(j\varpi)$.

Amplitudově fázová frekvenční charakteristika - je zobrazení frekvenčního přenosu v komplexní rovině. Nazývá se také Nyquistova křivka.

Logaritmické frekvenční charakteristiky - je grafické znázornění frekvenčního přenosu v logaritmických souřadnicích. Existují dva druhy těchto frekvenčních charakteristik:

- logaritmická fázová charakteristika

- logaritmická amplitudová charakteristika

Tyto logaritmické charakteristiky bývají označovány jako Bodeho Křivky

4 STABILITA REGULAČNÍHO OBVODU

Stabilita dynamického obvodu je schopnost vrátit se po vychýlení zpět do původního rovnovážného stavu. Toto vychýlení je vždy způsobeno nenulovými počátečními podmínkami.(A.M.Ljapunov ~ 1895). Z hlediska stability rozlišujeme regulační obvod stabilní, na mezi stability, nestabilní.

4.1 Kritéria stability

Kritéria stability LSDS umožňují rozhodnout o stabilitě uzavřeného regulačního obvodu bez výpočtu jeho kořenů. Tato kritéria dělíme na dvě skupiny.

4.1.1 Algebraická kritéria stability

Tato kritéria vycházejí z charakteristické rovnice dynamického systému, resp. z charakteristického polynomu dynamického systému. Pomocí těchto kritérií lze rozhodnout, zda systém je stabilní nebo není stabilní, ale nedávají informaci do jaké míry je systém tlumený. Kritéria nelze použít při vyšetřování stability systémů s dopravním zpožděním. Mezi algebraická kritéria stability patří:

- Hurwitzovo kritérium
- Routhovo-Schurovo kritérium

4.1.2 Geometrická kritéria stability

- Michajlovo-Leonhardovo kritérium
- Nyquistova kritérium

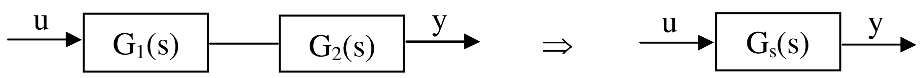
5 BLOKOVÁ ALGEBRA

5.1 Základní zapojení

Rozlišujeme sériové, paralelní, zpětnovazební (antiparalelní) zapojení.

5.1.1 Sériové zapojení

U sériového zapojení platí, že výsledný přenos je dán součinem jednotlivých sériově řazených přenosů.

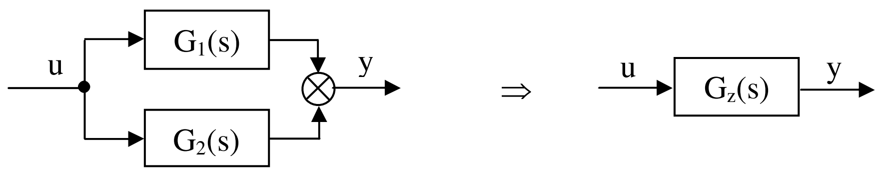


Obr. 2. Schéma sériového zapojení

$$G(s)=\frac{Y(s)}{U(s)}=G_1(s)\cdot G_2(s)$$

5.1.2 Paralelní zapojení

U paralelního zapojení je celkový přenos roven součtu jednotlivých paralelně řazených přenosů.

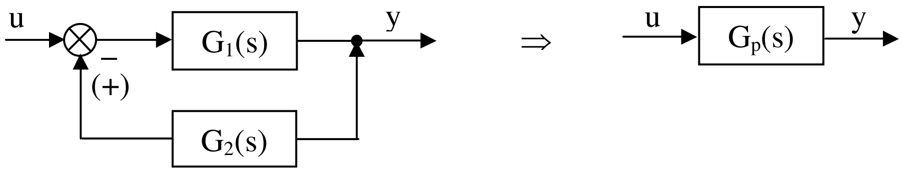


Obr. 3. Schéma paralelního zapojení

$$G_z(s)=\frac{Y(s)}{U(s)}=G_1(s)+G_2(s)$$

5.1.3 Zpětnovazební (antiparalelní) zapojení

U zpětnovazebního zapojení je výsledný přenos dán zlomkem, kdy v čitateli je přenos přímé větve a ve jmenovateli 1 ± součin přenosů v přímé a zpětnovazební větvi.



Obr. 4. Schéma zpětnovazebního zapojení

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{G_1(s)}{1 \pm G_1(s)G_2(s)}$$

Rozvětvené regulační obvody

Jednoduché jednorozměrové regulační obvody mohou splnit většinu běžných regulačních úkolů. Při vyšších požadavcích na přesnost a dynamiku regulace, hlavně u složitějších regulovaných soustav, jsou jejich možnosti omezené. Použitím rozvětvených obvodů se získají lepší dynamické i statické vlastnosti celého systému.

Používají se zejména následující typy rozvětvených regulačních obvodů:

a) s měřením poruchy

b) s pomocnou akční veličinou

c) s pomocnou řízenou veličinou

d) s kompenzací dopravního zpoždění – Smithův prediktor

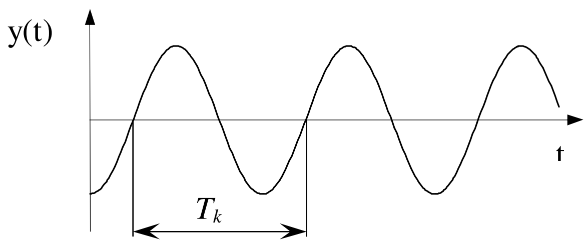
6 METODY NASTAVENÍ PID REGULÁTORŮ

Podle standardní literatury je PID přenos uvažován ve tvaru

$$G_R(s) = r_0 + \frac{r_{-1}}{s} + r_1 s \text{ nebo } G_R(s) = k_p\left(1 + \frac{1}{T_I s} + T_D s\right)$$

1. Ziegler – Nicholsova metoda kritického zesílení

Základní myšlenkou této metody je přivést regulační obvod na hranici stability. Toho se dosáhne použitím pouze proporcionální složky PID regulátoru ve zpětné vazbě. Integrační a derivační složky jsou vyřazeny. Zvyšuje se zesílení k_P (r_0), až k hodnotě kritického zesílení k_{Pk} (r_{0k}), a periodu kritických kmitů T (T_k), tak aby byl obvod na hranici stability. Podle použitého regulátoru vybereme vhodný vztah z tabulky a dosadíme získané hodnoty, čímž dostaneme parametry regulátoru.



Obr. 5. Určení T_k při r_{0k}

Hodnoty kritického zesílení a kritické periody kmitů se dají určit také jiným způsobem, a to vložením relé (Hägglund 1983 – autotuning) do zpětné vazby regulačního obvodu a následným výpočtem.

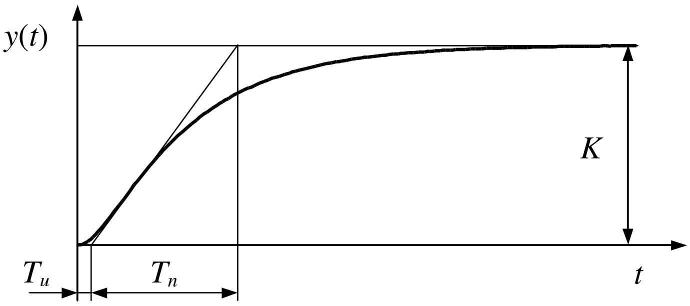
2. Nastavení z přechodové charakteristiky (aperiodického typu)

Z naměřené přechodové charakteristiky regulované soustavy odečteme hodnoty T_n, T_u, K.

kde: T_n … doba náběhu , T_u … doba průtahu , K … zesílení

Pomocí těchto hodnot vypočteme parametr γ podle vztahu

$$\gamma = \frac{T_n}{T_u}$$



Obr. 6. Určení parametrů K, T_u a T_n

Následně vybereme z tabulky vztah odpovídající příslušnému typu regulátoru, a dosazením získaných hodnot do tohoto vztahu dostaneme parametry regulátoru.

3. Cohen – Coonova metoda

Vychází se z tří-parametrového modelu

$$G_S(s) = \frac{K}{1+sT} e^{-\Theta s}$$

Tato metoda je navržena tak, že dává poměr tlumení 1/4. To znamená, že tato metoda návrhu regulátoru bude mít odezvu u druhého kmitu čtvrtinu první amplitudy.

Další metody:

- metoda vyváženého nastavení

- využití kritické stability pro návrh regulátorů

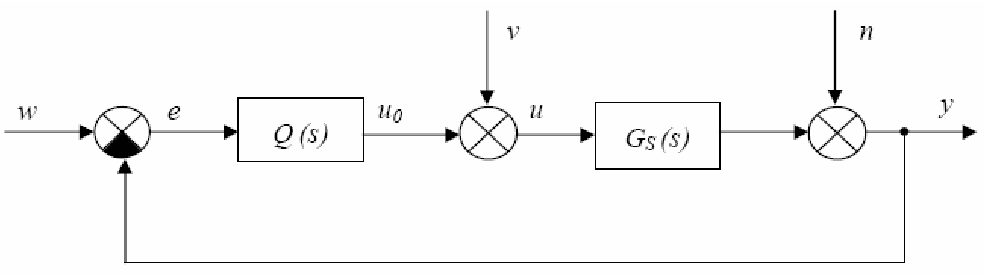
- Whiteleyho standardní tvary

Tyto metody jsou podrobně popsány v prezentacích

7 SYNTÉZA REGULÁTORŮ

1DOF konfigurace systému řízení

Systém s jedním stupněm volnosti (pouze se zpětnovazební části regulátoru)



Obr. 7. 1DOF konfigurace systému řízení

kde Q(s) – přenos regulátoru, G_S(s) – přenos soustavy, w – žádaná hodnota, v – porucha na vstupu, n – porucha na výstupu.

Přenos soustavy

$$G(s)=\frac{b(s)}{a(s)}$$

Přenos regulátoru

$$Q(s)=\frac{q(s)}{p(s)}$$

kde a(s), b(s) jsou nesoudělné polynomy u nichž je uvažováno, že $\deg b \le \deg a$, tzn. že přenos je ryzí, q(s) a p(s) jsou také nesoudělné polynomy.

Obecné požadavky na vlastnosti systému řízení

◇ stabilita systému řízení

zajišťuje zpětnovazební část regulátoru Q(s) daný řešením diofantické rovnice $ap + bq = d$

◇ vnitřní ryzost systému řízení

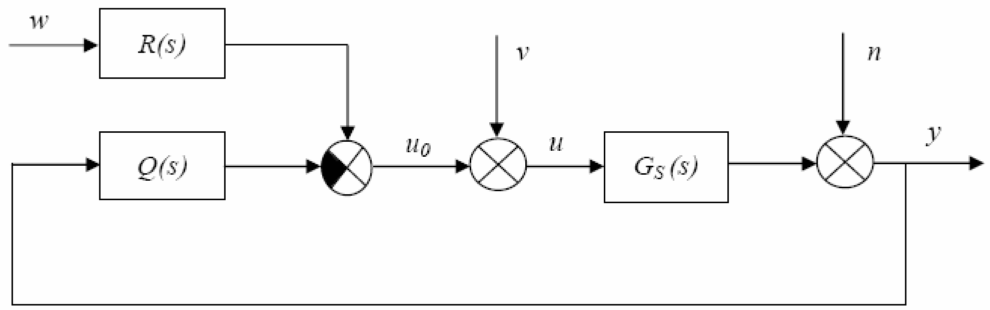
musí být splněna podmínka $deg\ q \le deg\ p$

◇ asymptotické sledování referenčního signálu a kompenzace poruch působících v systému

musí platit pro regulační odchylku $\lim_{t\to\infty}[e(t)] = \lim_{s\to 0}[s\cdot e(s)] = 0$

2DOF konfigurace systému řízení

Systém se dvěma stupni volnosti (s přímovazební i zpětnovazební částí)



Obr. 8. 2DOF konfigurace systému řízení

kde Q(s) – zpětnovazební část regulátoru, R(s) – přímovazební část regulátoru, G_S(s) – přenos soustavy, w – žádaná hodnota, v – porucha na vstupu, n – porucha na výstupu.

Přenos soustavy

$$G(s) = \frac{b(s)}{a(s)}$$

Přenos zpětnovazební a přímovazební části regulátoru

$$Q(s) = \frac{q(s)}{p(s)} \qquad R(s) = \frac{r(s)}{p(s)}$$

kde a(s), b(s) jsou nesoudělné polynomy u nichž je uvažováno, že $\deg b \leq \deg a$, tzn. že přenos je ryzí, q(s), p(s), r(s) a p(s) jsou také nesoudělné polynomy.

Obecné požadavky na vlastnosti systému řízení

◇ stabilita systému řízení

zajišťuje zpětnovazební část regulátoru Q(s) daný řešením diofantické rovnice $ap + bq = d$

◇ vnitřní ryzost systému řízení $\deg q \leq \deg p$

musí být splněna podmínka ryzosti přenosu zpětnovazebního regulátoru $\deg q \leq \deg p$ a podmínka ryzosti přenosu přímovazebního regulátoru $\deg r \leq \deg p$

◇ asymptotické sledování referenčního signálu a kompenzace poruch působících v systému

musí platit pro regulační odchylku $\lim_{t \to \infty}[e(t)] = \lim_{s \to 0}[s \cdot e(s)] = 0$

II. PRAKTICKÁ ČÁST

8 PREZENTACE

8.1. Úvod, Teorie systémů

Theory of Automatic Control

Department of Automation and Control Engineering

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Theory of Automatic Control

Content

- 1. System Theory
  - history, literature, system conception, classification of systems, mathematic models, feedback, mechanic models, electrics models, LCDS, differential equation, control system, SISO, MIMO
- 2. Laplace Transform
  - definition, properties, exploitation, patterns and pictures, dictionary LT, differential equation, transport delay
- 3. LCDS (Linear Continuous Dynamic System)
  - various descriptions, transfer function, differential equation, unit step response, impulse response, classification of LCDS, zero, pole, amplitude response

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Theory of Automatic Control

Content

- 4. Stability
  - definition, Lyapunov and BIBO stability, necessary and sufficient condition, first necessary condition, minimum and nonminimum phase
- 5. Block Diagram Algebra
  - feedback, control system – development and shapes, 1DOF, 2DOF, common control system, disturbance, sensitivity function
  - components, characteristics, quality of regulation process, Smith's predictor, saturation

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Theory of Automatic Control

Content

- 6. Methods of setting PID regulators
  - Ziegler - Nichols, Whiteley, Naslin, Cohen – Coon, balanced setting autotuning, relay control, identification and estimation of transfers, introduction to the nonlinear systems
- 7. Synthesis of controller
  - range, solid, diophantine equations
  - project regulators by the help of algebraic methods, polynomial synthesis 1. and 2. degree, pole placement, modification for transfer delay

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Theory of Automatic Control

Literature

- English literature :
  - Bishop, R.H. : Modern control system using Matlab and Simulink. Adison Wasley, Menlo Park, 1997
  - Mošcinski, J., Odonowski, Z. : Advanced Control with Matlab and Simulink, Ellis Horwood, London, 1995
  - Kuo, C. B. : Automatic Control Systems. Wiley, 2002

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Theory of Automatic Control

Literature

- Internet:
  - www.e-automatizace.cz
  - www.controlengcesko.com

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Theory of Automatic Control

Course description:

- The aim of this subject is assumption of knowledge and practice of identification, analysis and design of linear continuous dynamic systems. It is intend on principle of identification, model of disturbances and dynamic systems, control system structure, stability of linear control systems, time- and frequency-domain analysis and design. It includes the principles of the variable analysis and synthesis including observes. Theoretical and practical lessons use software support from MATLAB .

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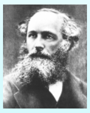
Theory of Automatic Control

History:

- J. Watt invented his steam engine in 1769, and this date marks the accepted beginning of the Industrial Revolution.
- problem associated with the steam engine is that of steam-pressure regulation in the boiler, for the steam that drives the engine should be at a constant pressure. In 1681 D. Papin invented a safety valve for a pressure cooker, and in 1707 he used it as a regulating device on his steam engine.
- J.C. Maxwell provided the first rigorous mathematical analysis of a feedback control system in 1868.



J.C. Maxwell

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Theory of Automatic Control

- we could call the period before about 1868 the *prehistory* of automatic control.
- we may call the period from 1868 to the early 1900's the *primitive period* of automatic control. It is standard to call the period from then until 1960 the *classical period*, and the period from 1960 through present times the *modern period*.
- In 1840, the British Astronomer Royal at Greenwich, G.B. Airy, developed a feedback device for pointing a telescope. His device was a speed control system which turned the telescope automatically to compensate for the earth's rotation, affording the ability to study a given star for an extended time.

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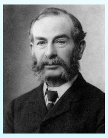
Theory of Automatic Control

- The early work in the mathematical analysis of control systems was in terms of differential equations. J.C. Maxwell analyzed the stability of Watt's fly ball governor [Maxwell 1868]. His technique was to linearize the differential equations of motion to find the *characteristic equation* of the system. He studied the effect of the system parameters on stability and showed that the system is stable if the roots of the characteristic equation have *negative real parts*. With the work of Maxwell we can say that the theory of control systems was firmly established.
- E.J. Routh provided a *numerical technique* for determining when a characteristic equation has stable roots [Routh 1877].



E.J. Routh

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Theory of Automatic Control

- The work of A.M. Lyapunov was seminal in control theory. He studied the stability of nonlinear differential equations using a generalized notion of energy in 1892 [Lyapunov 1893]. Unfortunately, though his work was applied and continued in Russia, the time was not ripe in the West for his elegant theory, and it remained unknown there until approximately 1960, when its importance was finally realized.
- The British engineer O. Heaviside invented operational calculus in 1892-1898. He studied the transient behavior of systems, introducing a notion equivalent to that of the *transfer function*.

A.M. Lyapunov

O. Heaviside

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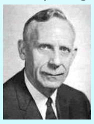
Theory of Automatic Control

- The mathematical analysis of control systems had heretofore been carried out using differential equations in the *time domain*. At Bell Telephone Laboratories during the 1920's and 1930's, the *frequency domain* approaches developed by P.-S. de Laplace (1749-1827), J. Fourier (1768-1830), A.L. Cauchy (1789-1857), and others were explored and used in communication systems.
- Regeneration Theory for the design of stable amplifiers was developed by H. Nyquist [1932]. He derived his *Nyquist stability criterion* based on the polar plot of a complex function. H.W. Bode in 1938 used the magnitude and phase *frequency response plots* of a complex function [Bode 1940].He investigated closed-loop stability using the notions of *gain and phase margin*.



H.W. Bode

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Theory of Automatic Control

- N. Minorsky [1922] introduced his three-term controller for the steering of ships, thereby becoming the first to use the *proportional-integral-derivative (PID)* controller. He considered nonlinear effects in the closed-loop system.
- R. Bellman [1957] applied *dynamic programming* to the optimal control of discrete-time systems, demonstrating that the natural direction for solving optimal control problems is *backwards in time.* His procedure resulted in closed-loop, generally nonlinear, feedback schemes.

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1.System Theory

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1.System theory

System – group of elements where exists relationship between them and environment at the same time

Systems: a) **substantial (real)** - engine, car, boat, boiler...
b) **abstract (exemplar)** - math functions, logic relationship, verbal formulations

Concrete abstract definition: R.E. Kalman ~ 1969
L.A. Zadeh ~ 1963, …

Simplified technical-economic system:

system **S** - entity set S={P,R,U,Y}

P - set of element
R - set of relations between elements
U - set of input magnitudes
Y - set of output magnitudes

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1.System theory

Control – purposeful incidence on object with the aim of ensure requested behaviour hereof object

Type of control: 1) **manual**
2) **automatic** - direct (without power supply)
- indirect (with power supply)

In term of way: 1) **continuous control**
2) **logical control**
3) **discrete control**

Automatic control – removing productive process dependence on physiological property of man

Automation – combination theoretical branches + engineering units

Control theory – Cybernetics - Norbert Wiener ~ 1948

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1.System theory

Cybernetics – Interdisciplinary discipline with using:
- math (analyse, probability,…)
- logic (automats, recognition,…)
- information (transfer, signal noise,…)

Cybernetics - theoretical
- technical

Cybernetics braches – system theory, information theory, theory of coding, theory of control, recognition figures, algorithm theory

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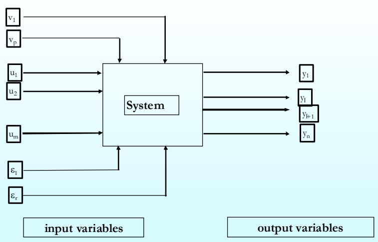
1.System theory

Variable classification:



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1.System theory

System classification in accordance with relations:

S < static / dynamic; linear / nonlinear; one-dimensional system / multivariable system; deterministic / stochastic

S < t-var. / t-inv.

Dynamic system < continuous / discrete

discrete < general / finite count of state system

finite count of state system < logic / automat

1.System theory

Examples:

$y = 4u_1 + 8u_2 + 16u_1^2 + 28$ ....static multivariable nonlinear t-invar. system

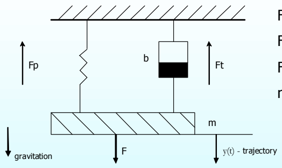
$y''(t) + 9y'(t) + 3y'(t) = u(t) + 6u'(t)$ ...dynamic linear t-invar. single – input / single – output system

Identification < Analytical / Experimental } Elicitation of mathematic model

1.System theory

Analytic mathematical models

1.Simple mechanical system with damping



F_p = c.y ... spring tension

F_t = b.y' ... power of steering dumper

F = u ... outer power

m, b, c ... constants

Solution: - Newton's low (balance of powers)

$$my''(t) = u(t) - by'(t) - cy(t)$$

1.System theory

- Transfer to differential equation second order

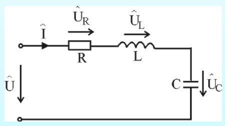
$$my''(t) + by'(t) + cy(t) = u(t)$$

- Gravitation field $u(t) = m.g$

- General solution of equation $y(t) = k_1 e^{-\alpha_1 t} + k_2 e^{-\alpha_2 t}$,

a1,2 are solving of $m.\alpha^2 + b.\alpha + c = 0$

2. RLC electric circuit



U (t) ...input voltage

Y (t) ...output voltage on condenser

I (t) ...current in circuit

1.System theory

1.Kirchoff's voltage low $u = u_R + u_C + u_L$

$$u(t) = L\frac{di(t)}{dt} + Ri(t) + \underbrace{\frac{1}{C}\int_0^t i(\tau)d\tau}_{y(t)}$$

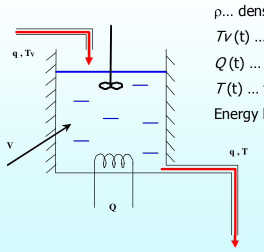
$$\left.\begin{aligned} y(t) &= \frac{1}{C}\int_0^t i(\tau)d\tau \\ y'(t) &= \frac{1}{C}i(t) \\ y''(t) &= \frac{1}{C}i'(t) \end{aligned}\right\} \quad u(t) = LCy''(t) + RCy'(t) + y(t)$$

Rem:1. again differential equation of second order

2. $\frac{dy(t)}{dt} = y'(t) = y(t)$

1.System theory

3. Hot-water boiler (flow heat exchanger with mixing)



q ... flow rate V ...capacity

ρ... density C_p ... heat capacity

Tv (t) ... input temperature - disturbance variable

Q (t) ... input energy – input variable

T (t) ... temperature – output variable

Energy budget

1.System theory

$$\underbrace{q\rho c_p T_V(t)}_{\text{input heat}} + \underbrace{Q(t)}_{\text{warm-up}} = \underbrace{q\rho c_p T(t)}_{\text{effluent heat}} + \underbrace{V\rho c_p \frac{dT(t)}{dt}}_{\text{stored heat}}$$

Equation editing: $\frac{V}{q}\frac{dT(t)}{dt} + T(t) = T_V(t) + \frac{1}{q\rho c_p}Q(t)$

Substitution: $y(t) = T(t)$ $Q(t) = u(t)$ $T_V(t) = v(t)$

Solution: $\tau y'(t) + y(t) = Ku(t) + v(t)$, $\tau = \frac{q}{v}$, $K = \frac{1}{q\rho c_p}$

Conclusion: 1. Different physical systems steer for same mathematical model ⟹ differential equation
2. Abstract mathematical model

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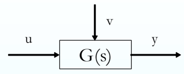
1.System theory

G(s) characterize differential equation ⟹ transfer



u (t) ... input variable
y (t) ... output variable
v (t) ... disturbance variable

3.What is linearity?

Operator L(u) = y is called linear <=> $L(\alpha u_1 + u_2) = \alpha L(u_1) + L(u_2)$

$y'(t) + 2y(t) = 5u(t)$
$y''(t) + 2y''(t) + 5y'(t) + 10y'(t) = 12u(t)$ } Linear

$y'(t) + 2[y(t)]^2 = 5u(t)$
$y''(t) + 2\ln y'(t) + 3y(t) = 10u(t)$
$y'(t) + 2y(t) = e^{-2u(t)}$ } Nonlinear

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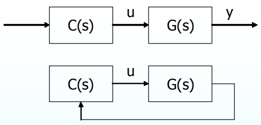
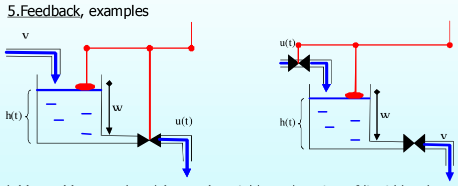
1.System theory

4.Control: Feedforward / Feedback



5.Feedback, examples



h (t) = y (t) ... regulated (output) variable – elevation of liquid level
w ... desired variable u (t) ... manipulated variable v ... disturbance

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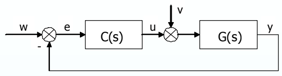
1.System theory

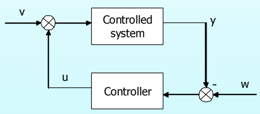
Abstraction



1.DOF structure

y ... controlled variable u ... manipulated variable
w ... desired variable v ... disturbance
$w - y = e$ ... control deviation

1.Classical structure



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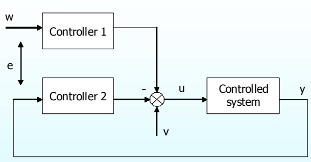
1.System theory

2.Modern structure



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8.2. Laplaceova transformace

2.Laplace Transforms

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1

2.Laplace transforms

Laplace transforms (**LT**) - is mathematical method, which makes it possible to easily solve continuous linear regulation problems.

Linear continuous dynamic system (LCDS)

SISO – Single input single output – one-dimensional

Description:

$$y^{(n)}(t)+a_{n-1}y^{(n-1)}(t)+....+a_1y'(t)+a_0y(t)=b_m u^{(m)}(t)+....+b_0u(t)$$

m<n is properness, causality system

1.Initial conditions aren't substantial for LCDS, but are important for explicit solution differential equation

2.How is solved differential equation - classical way = variation constant method

Example: $y'(t)+2y(t)=1 \qquad y(0)=0$

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2.Laplace transforms

a) **Homogeneous solution**

presumption: $y_h(t)=c_0\cdot e^{\lambda t}$

$$c_0\lambda e^{\lambda t}+c_0e^{\lambda t}=0 \qquad \Big/\cdot\frac{1}{c_0}\cdot e^{-\lambda t}$$

$$\lambda=-2\Rightarrow y_h(t)=e^{-2t}$$

b) **Inhomogeneous solution**

variation $c_0\to c(t)$

presumption $y(t)=c(t)e^{-2t}$

substitution $c'(t)e^{-2t}-2c(t)e^{-2t}+2c(t)e^{-2t}=1$

$$c'(t)=e^{2t}\Rightarrow c(t)=\frac{1}{2}e^{2t}+K$$

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2.Laplace transforms

solution $y(t)=\frac{1}{2}+Ke^{-2t}$

$$y(0)=0=\frac{1}{2}Ke^{-2.0}\Rightarrow K=-\frac{1}{2}$$

final solution $y(t)=\frac{1}{2}-\frac{1}{2}e^{-2t}$

Conclusion: This way is complicated and unacceptable from engineering aspect. Better solution is using Laplace transforms [1749 - 1827]

Laplace transforms (LT)

Main relation $f(t)\to F(s)=L\{f(t)\}=\int_0^\infty f(t)e^{-st}dt$

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2.Laplace transforms

condition of **f(t)** **a)** $f(t)=0$ **for** $t<0$

b) $|f(t)|\le Me^{-\lambda_0 t}$ **;** M,λ_0 **finite**

c) piecewise continuous

Inverse LT $f(t)=L^{-1}\{F(s)\}=\frac{1}{2\pi j}\oint F(s)e^{st}ds$

Rem.: 1. LT is mapping a set of real functions into a set of complex functions

2. The complex variable in the LT is in classical references indicated by a letter p.

3. Since to beginning 20.century is used LT for solving differential equation

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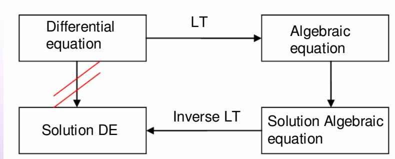
2.Laplace transforms



4. For direct and inverse LT is used dictionary

5. Exist other transforms, e.g. Fourier transforms

$$f(t)\to F(\varpi)=\int_{-\infty}^{+\infty}f(t)e^{-j\varpi t}dt$$

6. marking:
f(t), *g*(t), *h*(t) ... time function
F(s), *G*(s), *H*(s) ...LT images, complex function

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2.Laplace transforms

Laplace transforms properties

1. Differentiation $L\{f'(t)\} = sF(s) - f(0)\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$

$L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - \ldots - s f^{(n-1)}(0) - f^{(n-1)}(0)$

2. Primary function $L\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}F(s)$

3. Initial and final value theorem $f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

$f(0) = \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$

4. Shift theorem $L\{F(t-\tau)\} = e^{-\tau s}F(s)$

2.Laplace transforms

5. Improper integral $L\left\{\int_0^\infty f(t)dt\right\} = \lim_{s\to 0} F(s)$

6. Convolution theorem $L\left\{\int_0^\infty f(\tau)g(t-\tau)dt\right\} = F(s)\cdot G(s)$

7. Linearity $L\{\alpha f(t) + g(t)\} = \alpha F(s) + G(s)$

Difference between linear and nonlinear function

$y''(t) + 3y'(t) + 5y(t) = 8u(t)$ ...linear

$y'(t) + 2[y(t)]^2 = u(t)$ ...nonlinear

$y''(t) + 2y'(t)\cdot y(t) = 2u(t)$ ...nonlinear

2.Laplace transforms

Most often used pattern of LT dictionary:

No.	Time domain function	Laplace transform
1	δ (Dirac delta function)	1
2	1 (unit step function)	$\frac{1}{s}$
3	t (linear ramp)	$\frac{1}{s^2}$
4	e^{-at}	$\frac{1}{s+a}$
5	t^n	$\frac{n!}{s^{n+1}}$
6	sin ωt	$\frac{\omega}{s^2+\omega^2}$

2.Laplace transforms

Most often used pattern of LT dictionary:

No.	Time domain function	Laplace transform
7	cos ωt	$\frac{s}{s^2+\omega^2}$
8	$\frac{1}{a}\left(1 - e^{-at}\right)$	$\frac{1}{s(s+a)}$
9	e^{-at} sin ωt	$\frac{\omega}{(s+a)^2+\omega^2}$
10	e^{-at} cos ωt	$\frac{s+a}{(s+a)^2+\omega^2}$
11	$t\cdot e^{-at}$	$\frac{1}{(s+a)^2}$
12	$e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$

2.Laplace transforms

Most often used pattern of LT dictionary:

No.	Time domain function	Laplace transform
13	$t^{n-1}\cdot e^{-at}$	$\frac{(n-1)!}{(s+a)^n}$
14	$e^{-at}(1-e^{-at})$	$\frac{s}{(s+a)^2}$
15	1-sin ωt	$\frac{\omega^2}{s(s^2+\omega^2)}$
16	1-cos ωt	$\frac{s^2+\omega^2-s\omega}{s(s^2+\omega^2)}$

2.Laplace transforms

Examples:

1) $f(t) = 1 \Rightarrow F(s) = \int_0^\infty e^{-st}dt = \frac{1}{s}$

2) $f(t) = t \Rightarrow F(s) = L\left\{\int_0^t 1 d\tau\right\} = \frac{1}{s}\cdot\frac{1}{s} = \frac{1}{s^2}$

3) $f(t) = e^{-at} \Rightarrow F(s) = \int_0^\infty e^{-(s+a)t}dt = \frac{1}{s+a}$

$\int_0^\infty e^{-2t}dt = \lim_{s\to 0} F(s) = \frac{1}{2}\ldots$

2.Laplace transforms

Example of using **LT dictionary** during solving differential equation:

1) $y'(t)+2y(t)=1 \qquad y(0)=0$

LT: $sY(s)-Y(0)+2Y(s)=\frac{1}{s}$

$$Y(s)=\frac{1}{s(s+2)}=\frac{0,5}{s}-\frac{0,5}{s+2}\Rightarrow y(t)=0,5-e^{-2t}$$

2) $y''(t)+9y(t)=0$

a) $y(0)=1 \qquad y'(0)=0$

b) $y(0)=0 \qquad y'(0)=1$

$$s^2Y(s)-sy(0)-y'(0)+9Y(s)=0$$

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2.Laplace transforms

$$(s^2+9)Y(s)=sy(0)+y'(0)$$

$$Y(s)=\frac{s}{(s^2+9)}\Rightarrow y(t)=\cos 3t$$

$$Y(s)=\frac{1}{(s^2+9)}=\frac{1}{3}\frac{3}{(s^2+9)}\Rightarrow y(t)=\frac{1}{3}\sin 3t$$

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2.Laplace transforms

Inverse Laplace transforms

Residues theorem

Over all poles

Direct: $f(t)\rightarrow F(s)=L\{f(t)\}=\int_0^{\infty} f(t)e^{-st}dt$

Inverse: $F(s)\rightarrow f(t)=L^{-1}\{F(s)\}=\frac{1}{2\pi j}\oint F(s)e^{st}ds=\sum_i res\left[F(s_i)e^{s_it}\right]$

Residuum of complex function – coefficient in Laurent's function expansion at (-1) power

Residues calculation

1) One-multiple pole $\quad res[F(s)_i]=\lim_{s\to s_i}[(s-s_i)F(s)]$

2) N-multiple pole $\quad res[F(s_i)]=\frac{1}{(n-1)!}\lim\left[\frac{d^{n+1}}{ds^{n-1}}(s-s_i)^nF(s)\right]$

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2.Laplace transforms

Heaviside - partial fraction expansion by backward LT

a) One-multiple pole

$$G(s)=\frac{b_ms^n+....+b_0}{(s-s_1)....(s-s)}=\frac{A_1}{s-s_1}+\frac{A_2}{s-s_2}+....+\frac{A_n}{s-s_n}$$

$$A_i=\lim_{s\to s_i}[(s-s_i)G(s)]$$

b) K-multiple pole

$$B_k=\left[(s-s_2)^kG(s)\right]_{s-s_1}$$

$$B_{k-1}=\left[\frac{1}{1!}\frac{d}{d_s}(s-s_1)^kG(s)\right]_{s=s_i} \quad ....B_1=\left[\frac{1}{(k-1)!}\frac{d}{ds^{k+1}}(s-s_1)^kG(s)\right]_{s=s_1}$$

$$G(s)=\frac{b_ms^m+...+b_0}{(s-s_1)^k(s-s_2)...(s-s_i)}=\frac{B_1}{(s-s_1)}+\frac{B_2}{(s-s_1)^2}+..+\frac{B_k}{(s-s_1)^k}+\frac{A_2}{s-s_2}+..+\frac{A_i}{s-s_i}$$

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2.Laplace transforms

Heaviside development – can be used also in cases, when some of the roots are complex conjugate.

In contrast to **method of undetermined coefficients** the function have got different form

Indeterminate coefficient

$$F(s)=\frac{1}{(s+2)(s^2+2s+5)}=\frac{A}{s+2}+\frac{Bs+C}{s^2+2s+5}$$

Heaviside

$$F(s)=\frac{1}{(s+2)(s^2+2s+5)}=\frac{A}{s+2}+\frac{A_2}{s+1-2i}+\frac{A_3}{s+1+2i}$$

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2.Laplace transforms

Example:

1) $F(s)=\frac{1}{(s+1)(s+2)s}$

2) $F(s)=\frac{1}{(s+1)(s+2)s}=\frac{A_1}{s}+\frac{A_2}{s+1}+\frac{A_3}{s+2}$

3) $A_1=\left[\frac{1}{(s+1)(s+2)}\right]_{s=0}=\frac{1}{2}$

$$A_2=\left[\frac{1}{s(s+2)}\right]_{s=-1}=-1$$

$$A_3=\left[\frac{1}{s(s+1)}\right]_{s=-2}=\frac{1}{2}$$

$$f\{t\}=L^{-1}\{F(s)\}=L^{-1}\left\{\frac{\frac{1}{2}}{s}-\frac{1}{s+1}+\frac{\frac{1}{2}}{s+2}\right\}=\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}$$

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8.3. Lineární spojité dynamické systémy

3.Linear Continuous Dynamic Systems

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- LCDS – Linear continuous dynamic system
- SISO – Single input – single output
  one - dimensional system
- MIMO – Multi input – multi output
  multidimensional system

Linear continuous dynamic system (LCDS)

Differential equation

$$y^{(n)}(t)+a_{n-1}y^{(n-1)}(t)+....+a_1y'(t)+a_0y(t)=b_mu^{(m)}(t)+....+b_0u(t)$$

m<n is properness, causality system

Transfer:

Transfer = Transfer function is the Laplace's images rate output variable to input variable at zero initial conditions.

Description LCDS

External – differential equation
- laplace transform
- zero and pole position
- unit step response
- impulse response
- frequency response
- frequency characteristic

Internal – state space description

Remark:

1) Is it fractional rational function, multinominal over multinominal

$$G(s)=\frac{Y(s)}{U(s)}=\frac{b_ms^m+...+b_0}{s^n+a_{n-1}s^{(n-1)}+...+a_1s+a_0}$$

2) Transfer and Laplace's image function have identical form:

Example: $\frac{1}{s+2}$ is image e^{-2t} and at the same time transfer of differential equation $y'(t)+2y(t)=u(t)$

3) Examples of describing function and differential function

$$y''(t)+3y'(t)+=2y(t)=5u(t)+u'(t)\Rightarrow G(s)=\frac{s+5}{s^2+3s+2}$$

$$G(s)=\frac{1}{s(s+1)}\Rightarrow y''(t)+y'(t)=u(t)$$

4) Transfer is important only for linear system

5) System order - order n

- relative order n – m

Zeros and poles LCDS

Zeros – roots of numerator

Poles – roots of denominator

$$G(s)=\frac{s+1}{(s+2)(s+3)}$$

Poles: p1= -2, p2= -3

Zeros: n1= -1, n2= 8

1.Poles of the transfer function was understand as time constant

$$\frac{1}{(s+2)(s+3)}=\frac{\frac{1}{6}}{(0{,}5s+1)\left(\frac{1}{3}s+1\right)}$$

generally $G(s)=\frac{b_0}{(T_1s+1)(T_2s+1)...(T_ns+1)}$

3.Linear continuous dynamic system

2. Poles determine about system stability, zeros represent minimum phase

3. Modern Theory of automatic control says that poles and zeros is same number, and it *n* (order of system). Remaining zeros are in complex infinite.

Impulse and Unit step function

Definition:

Impulse function - is response to Dirac delta function at zero initial conditions

$i(t) \rightarrow I(s)$ Impulse function

Unit step function - is response to unit step at zero initial conditions

$h(t) \rightarrow H(s)$ Unit step function

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3.Linear continuous dynamic system

1. $I(s) = G(s)$ and $i(t) = L^{-1}\{G(s)\}$

Image of Impulse function is same as system transfer

2. $G(s) = \dfrac{H(s)}{\frac{1}{s}} \Rightarrow H(s) = \dfrac{G(s)}{s} \Rightarrow h(t) = L^{-1}\left\{\dfrac{G(s)}{s}\right\}$

3. $I(s) = \dfrac{H(s)}{\frac{1}{s}} \Rightarrow I(s) = sH(s) \Rightarrow i(t) = \dfrac{dh(t)}{dt} = h'(t)$

Impulse function is differentiation of unit step function

Unit step function is integral of impulse function $h(t) = \int_0^t i(\tau)d\tau$

4. $h(\infty) = \lim_{t\to\infty} h(t) = \lim_{s\to 0} s\dfrac{G(s)}{s} = \lim_{s\to 0} G(s)$

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3.Linear continuous dynamic system

Example:

Unit step function of transfer $G(s) = \dfrac{1}{(s+1)(s+2)}$ converge to 0,5

5. If *u*(t) is arbitrary input function to LCDS, so output is:

$$y(t) = \int_0^\infty i(\tau)u(t-\tau)d\tau \Leftrightarrow Y(s) = G(s)U(s)$$

Remark: 1.MATLAB commands IMPULSE ([1],[1 3 2])

STEP ([1],[1 3 2])

2.Unit step function of 1st order system $\dfrac{b_0}{s+a_p}$ doesn't have any flex point

Impulse function doesn't have extreme

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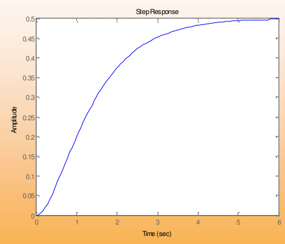
3.Linear continuous dynamic system

3.Unit step function $\dfrac{b_0}{(s+a_0)^n}$ is in the form:

High of flex point is proportional to order of system



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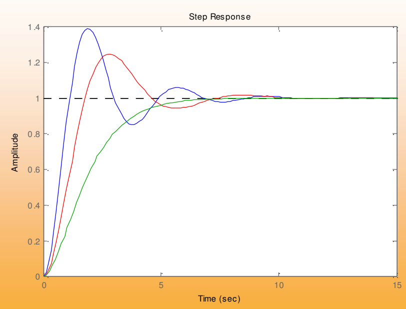
3.Linear continuous dynamic system

4. Typical unit step response of higher order



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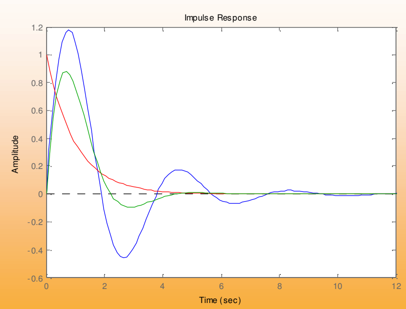
3.Linear continuous dynamic system

5. Typical impulse response of higher order



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3.Linear continuous dynamic system

Classification of LCDS by transfers:

a) Proportional $|h(\infty)| < \infty$ $G(s) = \frac{b(s)}{a(s)}$ $a \neq 0$

b) Derivative $|h(\infty)| = 0$ $G(s) = s^r \frac{b(s)}{a(s)}$ $dg\ a > dg\ b$

c) Integrative $h(\infty) = \infty$ $G(s) = \frac{b(s)}{s^k a(s)}$ $dg\ a > dg\ b$ or don't exist

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3.Linear continuous dynamic system

Frequency response

u(t)=$u_0$sin? t → G(s) → y(t)=$y_0$sin(? t+f)

What happens with harmonic signal after passage through the LCDS?

Without changes ? , angular speed $\varpi = 2\pi f = \frac{2\pi}{T}$

With changes: - y_0 amplitude

f shift phase

Marking in complex numbers:

$$\left.\begin{aligned} e^{j\varpi t} &= \cos\varpi t + j\sin\varpi t \\ e^{-j\varpi t} &= \cos\varpi t - j\sin\varpi t \end{aligned}\right\} \Rightarrow \begin{aligned} \sin\varpi t &= \frac{1}{2j}\left[e^{j\varpi t} - e^{-j\varpi t}\right] \\ \cos\varpi t &= \frac{1}{2}\left[e^{j\varpi t} + e^{-j\varpi t}\right] \end{aligned}$$

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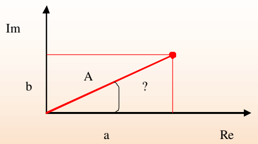
3.Linear continuous dynamic system



$a + jb = A(\cos\alpha + j\sin\alpha) = Ae^{j\alpha}$

$A = \sqrt{a^2 + b^2}$ $\alpha = arctg\frac{b}{a}$

Frequency response

$$G(j\varpi) = \frac{G(s)}{s} = j\varpi = \frac{b_m(j\varpi)^m + \dots b_0}{(j\varpi)^n + a_{n-1}(j\varpi)^{n-1} + \dots + a_0}$$

Frequency response is the Fourier's images rate (input and harmonic signal to output) in the course of zero initial conditions

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3.Linear continuous dynamic system

Remark: $G(j\varpi) = A(\varpi)e^{j\varphi(\varpi)} = \mathrm{Re}(\varpi) + j\,\mathrm{Im}(\varpi)$

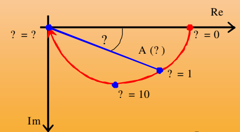
amplitude – phase frequency response

$G(j\varpi) = \int_0^\infty i(\tau)e^{-j\varpi\tau}d\tau$ impulse function

Amplitude – phase frequency response

Graphic representation of the frequency response – Nyquist curve

$$\frac{1}{2s+1} \to G(j\varpi) = \frac{1}{1+2j\varpi}\cdot\frac{1-2j\varpi}{1-2j\varpi} = \frac{1}{1+4\varpi^2}(1-2j\varpi) = \underbrace{\frac{1}{1+4\varpi^2}}_{\mathrm{Re}(?)} + j\underbrace{\frac{(-2\varpi)}{1+4\varpi^2}}_{\mathrm{Im}(?)}$$



$A(\varpi) = \sqrt{\mathrm{Re}^2(\varpi) + \mathrm{Im}^2(\varpi)}$

$\varphi(\varpi) = arctg\frac{\mathrm{Im}(\varpi)}{\mathrm{Re}(\varpi)}$

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3.Linear continuous dynamic system

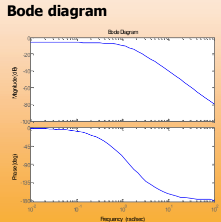
Logarithmic frequency response

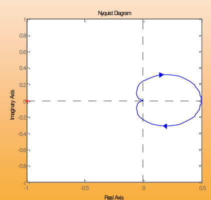
Graphic representation of A(?), f (?) in decimal logarithmic coordinates + amplitude in dB (decibel)

Example: $\frac{1}{s^2 + 3s + 2}$ $A[\mathrm{dB}] = 20\log A = 20\log\frac{y_0}{u_0}$

Nyquist ? ? <0,8 >

Bode diagram





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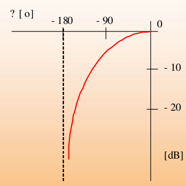
3.Linear continuous dynamic system

Remark:1. What is the meaning of 20 dB? Is concerned about 10 multiple of amplitude

$\frac{y_0}{u_0} = \frac{1}{10} \Rightarrow A[dB] = 20\log\frac{1}{10} = -20[dB]$



2. Nichols plot A(?) vs. F(?)

3. MATLAB commands

step ([1],[1 3 2])
impulse ([1],[1 3 2])
nyquist ([1],[1 3 2])
bode ([1],[1 3 2])
nichols ([1],[1 3 2])

4.Nonminimal phase system – zeros on the right side

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3.Linear continuous dynamic system

Systems with transport delay

Transport delay – disagreeable part of many systems

- is caused by dead time

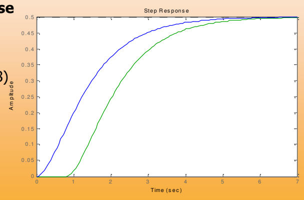
$y'(t)+3y(t)=2\underbrace{u(t-5)}$

Transport delay

Transfer function $G(s)=\dfrac{2e^{-5s}}{(s+3)}$

a)Influence on unit step response

MATLAB command
tr_function=tf([1],[1 3 2],'idelay',0.8)



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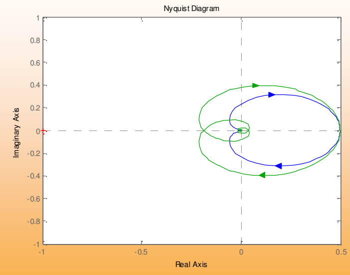
3.Linear continuous dynamic system

b) Influence on Nyquist plot



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8.4. Stabilita

4.Stability

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3.Stability

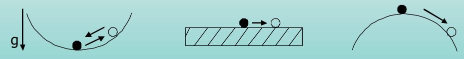
LCDS Stability

A.M. Lyapunov (~1895)

Stability of dynamic system is ability return to original condition after deflection. This deflection is caused by nonzero initial conditions.

Classical illustration

Ball in gravitation field



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3.Stability

Remark:

1) Ljapunov stability is independent of input variable *u*(t), it's property of left side differential equation.

$$y^{(n)}(t)+a_{n-1}y^{(n-1)}(t)+\ldots+a_0 y(t)=0$$

2) BIBO stability (Bounded input – Bounded output) – Bounded input generates bounded output. This stability is stricter. Ljapunov stability results from BIBO, opposite no.

LCDS stability: LCDS is stable ⇔ all roots of transfer function denominator (poles) lies in the left side of complex plane

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3.Stability

Remark:

1) Polynomial with roots in left side of complex plane is called stable.

2) Imaginary axis is stability limit, poles laying on it represent unstable system.

3) Left half is responded of modes $e^{p_i t}$,which converge to 0 only for negative parts of pi.

Necessary condition of Stability

Polynomial $a_n s^n + a_{n-1}s^{n-1}+\ldots+a_1 s + a_0$ is stable , then $a_i > 0$

Remark: 1) Polynomial with only one negative coefficient is unstable even.

2) Polynomial with all positive coefficient do not have to be stable.

$$s^3+0{,}5s^2+0{,}5s+1=(s+1)(s^2-0{,}5s+1)$$

roots: $p_1=-1 \quad p_{2,3}=0{,}25\pm\frac{\sqrt{3{,}75}}{2}$ ⇒ polynomial is unstable

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3.Stability

3) Necessary condition is the sufficient condition at the same time for polynomial the 1. and 2. order.

4) For determining position of roots is just necessary examine their real parts

Stability criteria

Algebraic approach
- Routh – Schure criterion
- Hurwitz criterion

Geometric approach
- Nyquist criterion
- Michailov-Leonard criterion

Stability limit ≡ Imaginary axis

Poles lies on imaginary axis ? polynomial is unstable

Special position is pole in zero ? integrators

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3.Stability

Routh – Schur criterion

$a_n s^n + a_{n-1}s^{n-1}+\ldots+a_1 s + a_0$

$$a_n \quad a_{n-1} \quad a_{n-2} \quad \ldots \quad a_2 \quad a_1 \quad a_0$$
$$a_{n-1} \quad a_{n-3} \quad a_1 \qquad -\frac{a_0}{a_{n-1}}$$
$$0 \quad a^1_{n-1} \quad a^1_{n-2} \quad \ldots \quad a^1_2 \quad a^1_1 \quad a^1_0$$
$$a^1_{n-2} \quad a^1_0 \qquad -\frac{a^1_{n-1}}{a^1_{n-2}}$$
$$0 \quad a^2_{n-2} \quad \ldots \quad a^2_2 \quad a^2_1 \quad a^2_0$$
$$\vdots$$
$$a^k_2 \quad a^k_1 \quad a^k_0$$

Polynomial is stable ⇔ last three coefficient are positive

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3.Stability

Remark:

1) Whenever is found any negative coefficient during Routh – Schure calculation system is unstable.

2) Zero value of coefficient indicate stability limit.

Hurwitz criterion

Matrix: $n \times n$ $\quad H_n = \begin{bmatrix} a_{n-1} & a_{n-3} & \dots & 0 \\ a_n & a_{n-2} & \dots & 0 \\ 0 & a_{n-1} & \dots & 0 \\ 0 & a_n & \dots & 0 \\ & & \ddots & a_0 \end{bmatrix}$

Polynomial is stable ⇔ all head subdeterminants are positive

Remark: Edward John Routh [1831-1907] – Canada, Great Britain

Adolf Hurwitz [1859-1919] – Switzerland

Isaac Schur [1875-1914] – Byelorussia, Palestine

3.Stability

Example: $s^4 + 2{,}5s^3 + 1{,}5s^2 + 2s + 2$

1	2,5	1,5	2	2	
2,5		2			$/ -\frac{1}{2,5}$
0	2,5	0,7	2	2	
	0,7		2		$/ -\frac{2,5}{0,7}$
	0	0,7	$-\frac{36}{7}$	2	

$$H_4 = \begin{bmatrix} 2{,}5 & 5 & 0 & 0 \\ 1 & 1{,}5 & 2 & 0 \\ 0 & 2{,}5 & 2 & 0 \\ 0 & 1 & 1{,}5 & 2 \end{bmatrix}$$

? 1 = 2,5

? 2 = 3,75 – 2 > 0

? 3 = 7,5 – 12,5 – 4 < 0

Unstable

3.Stability

Mikhailov – Leonard criterion

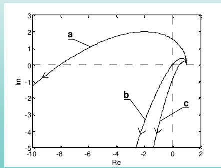
$a(s) = a_n s^n + \dots + a_1 s + a_0$

Mikhailov's curve construction $\quad a(j\varpi) = a(s)\big/_{s = j\varpi} \quad pro \ \ \varpi \in <0;\infty)$

Criterion:

Polynomial is stable ⇔ Mikhailov's curve is running in positive sense of rotation, as many quadrant as degree of polynomial

$s^3 + a_2 s^2 + a_1 s + a_0$



Remark:

1) a(j?) is reminiscent Niquist's curve

2) Zero exclusion theorem – robust stability

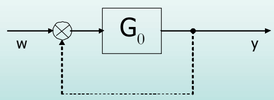
Mikhailov's curve behaviour for different polynomial

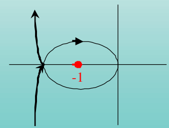
a) stable b) stability limit c) unstable

3.Stability

Nyquist criterion

Answer to question: How we can find from open-loop, whether closed loop will be stable.





Criterion:

Closed loop is stable ⇔ if G_0(j?) for ? ∈(-8 ,+8) is running in positive sense of rotation as many time the point (-1,0) as G_0 has got unstable poles.

Point (-1,0) meaning $K_{w,y} = \dfrac{G_0}{1+G_0}$

Usually in control system $G_0 = G \cdot C$

3.Stability

Meierov criterion

Gives answer to periodicity, but no stability

Characteristic feedback polynomial:

$c = ap + bq$ … b/a is controlled system

… p/q is controller

Criterion:

$c(s) = c_n s^n + c_{n-1} s^{n-1} + \dots + c_1 s + c_0$ Characteristic polynomial

Characteristic polynomial generates aperiodic control process ⇔ all coefficients in Routh – Schur's scheme are positive for sequence.

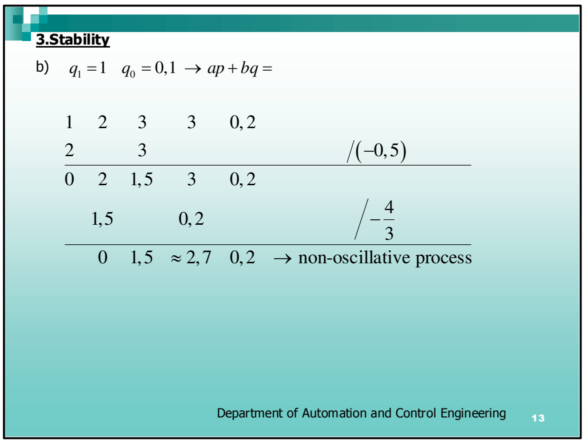
$c_n \ \ nc_n \ \ c_{n-1} \ \ (n-1)c_{n-1} \ \dots . \ c_2 \ \ 2c_2 \ \ c_1 \ \ 1c_1 \ \ c_0$

3.Stability

Example: $G(s) = \dfrac{2}{(s+1)} \qquad C(s) = \dfrac{q_1 s + q_0}{s}$

a) $q_1 = 0{,}1 \quad q_0 = 1 \rightarrow ap + bq = s^2 + 1{,}2s + 2$ … is stable

1	2	1,2	1,1	2	
2		1,1			$/(-0{,}5)$
0	2	0,55	1,1	1	
	0,55		1		$/ -\frac{2}{0,55} = -3{,}6$
	0	0,55	–2,5	1	→ oscillative process


3.Stability
b) q1 = 1 q0 = 0,1 → ap + bq =
1 2 3 3 0,2
2 3 /(−0,5)
0 2 1,5 3 0,2
1,5 0,2 / −4/3
0 1,5 ≈ 2,7 0,2 → non-oscillative process
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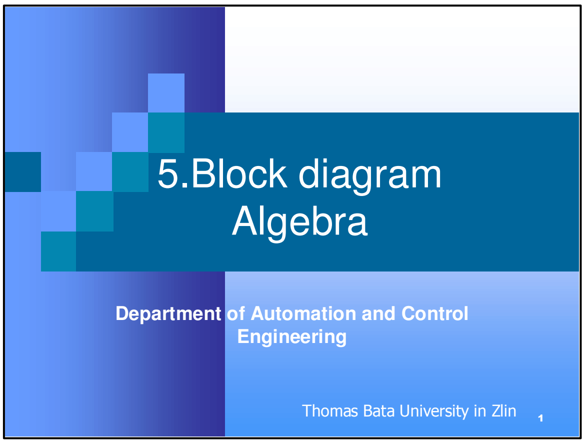
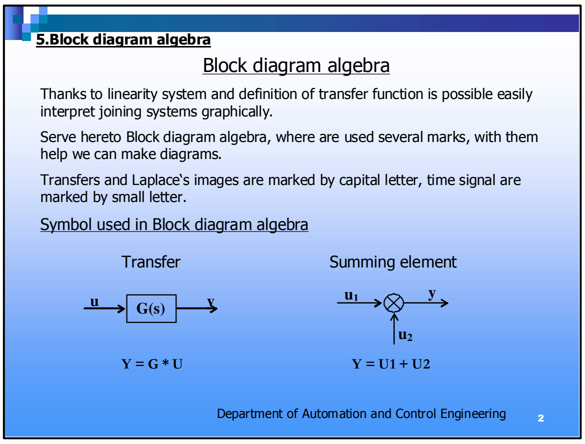
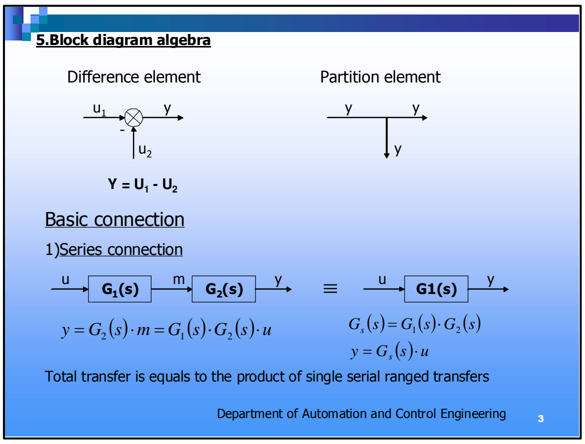
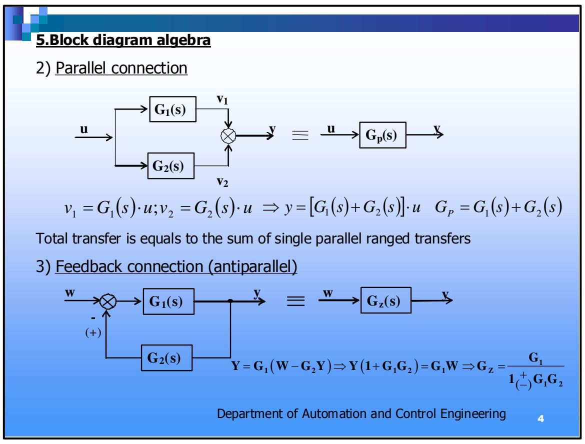
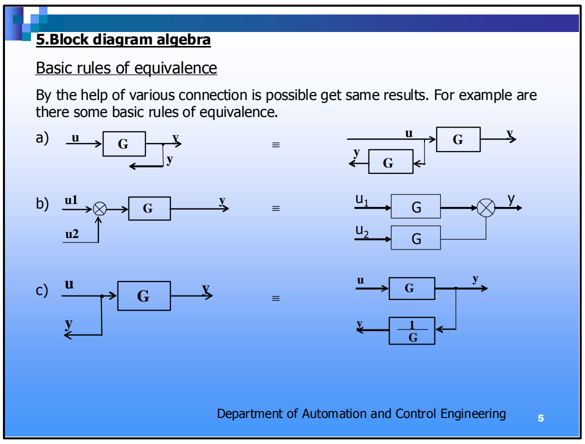
8.5. Bloková algebra

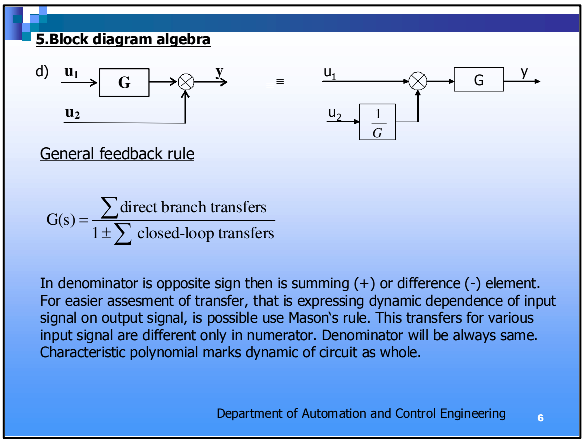








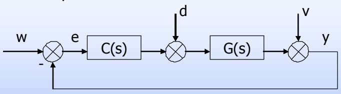




5.Block diagram algebra

Branched control system

Basic control system



This basic control system don't have to be always proper for many problems of control systems. We are using also other more complex feedback system.

Branched control system:

a) with disturbance measurement

b) with instrumental manipulated variable

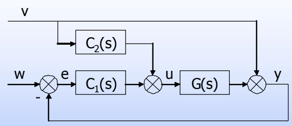
c) with instrumental control variable

d) with compensation of transport delay – Smith's predictor

5.Block diagram algebra

Control system with disturbance measurement

Results from block diagram algebra



control system scheme:

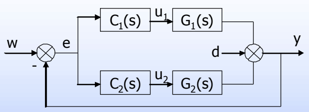
$$Y = V + G[-R2V + R1(W-Y)]$$

$$Y = \frac{GR_1}{1+GR_1}W + \frac{1-GR_2}{1+GR_1}V \Rightarrow 1 - GR_2 = 0$$

equality $\mathbf{GR_2 = 1}$ is physical unrealizable, is possible get near this number only

5.Block diagram algebra

Control system with instrumental manipulated variable



control system scheme:

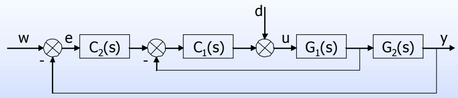
$$Y = D + R_1G_1(W-Y) + R_2G_2(W-Y)$$

$$Y = \frac{R_1G_1 + R_2G_2}{1 + R_1G_1 + R_2G_2}W + \frac{1}{1 + R_1G_1 + R_2G_2}D$$

stability increase

5.Block diagram algebra

Control system with instrumental control variable – self aligning control



control system scheme:

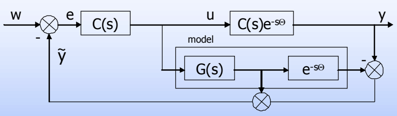
$$K_{W/Y} = \frac{G_1G_2R_1R_2}{1+G_1G_2R_1R_2 + G_2R_2} \qquad K_{y/d} = \frac{G_1G_2}{1+G_1G_2R_1R_2 + G_1G_2}$$

This transfer is possible influence by controller R1,R2

5.Block diagram algebra

Control system with compensation of transport delay – Smith's predictor

This connection makes possible to control systems with long transport delay



control system scheme:

Controller *R*(s) send out actuating variable as if the *G*(s) would be control without transport delay.

$$\tilde{y} = G(s)u + G(s)e^{-\Theta s}u - G(s)e^{-\Theta s}u = G(s)u$$

5.Block diagram algebra

If is supposed same transfer of model and real controlled system, then is valid this resultant transfer function.

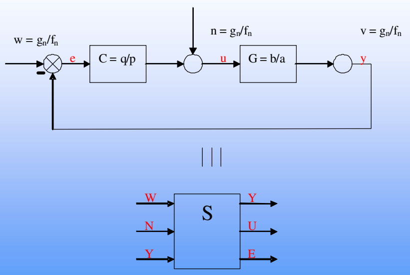
$$K_{W/Y}(s) = \frac{G(s)R(s)}{1+G(s)R(s)} \cdot e^{-\Theta s}$$

Characteristic equation $1 + G(s)R(s) = 0$

Is same as close-loop transfer function without transport delay. Smith's predictor conduce to creating all department of control system, so-called Internal Models Controllers (IMC).

5.Block diagram algebra

General control system



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5.Block diagram algebra

Analyse

$$Y = V + G[N + C(W - Y)] \longrightarrow Y(1 + CG) = GCW + GN + V$$

$$U = N + C[W - Y - GU] \longrightarrow U(1 + CG) = CW + N - CV$$

$$E = W - V - G(N + CE) \longrightarrow E(1 + CG) = W - GN - V$$

from it

$$\begin{pmatrix} Y \\ U \\ E \end{pmatrix} = \frac{1}{1+CG}\begin{pmatrix} CG & G & 1 \\ C & 1 & -C \\ 1 & -G & -1 \end{pmatrix}\begin{pmatrix} W \\ N \\ V \end{pmatrix} = \frac{1}{\underbrace{ap+bq}}\begin{pmatrix} bq & bp & ap \\ aq & ap & -aq \\ ap & -bp & ap \end{pmatrix}\begin{pmatrix} W \\ N \\ V \end{pmatrix}$$

Characteristic polynomial

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5.Block diagram algebra

Meaning of sensitivity function

Most important transfer $K_{W/Y} = \dfrac{CG}{1+CG}$

How is change K_{wy} on change ?G?

$$\lim_{\Delta G \to 0} \frac{\frac{\Delta K_{wy}}{K_{wy}}}{\frac{\Delta G}{G}} = \frac{G}{K_{wy}}\frac{dK_{wy}}{dG} = \frac{G}{\frac{CG}{1+CG}}\frac{C(1+CG) - CGC}{(1+CG)^2} = \frac{1}{1+CG} = K_{we} = \frac{ap}{ap+bq} = S$$

Disturbance transfer

$$K_{VY} = \frac{1}{1+CG}$$

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5.Block diagram algebra

Analytic:

$$\lim_{\Delta G \to 0} \frac{\frac{\Delta K_{vy}}{K_{vy}}}{\frac{\Delta G}{G}} = \frac{G}{\frac{1}{1+CG}}\frac{dK_{vy}}{dG} = \frac{GC}{1+CG} = K_{wy} = T$$

complementary sensitivity function

Remark:

1) $T + S = \dfrac{GC}{1+GC} + \dfrac{1}{1+GC} = \dfrac{GC+1}{1+GC} = 1$ complex functions

2) S and T cannot be done small at the same time. Controller cannot be done insensitive circuit in face of disturbance and reference signal at the same time.

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8.6. Metody nastavení PID regulátorů

6.Methods of setting PID controllers

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6.Methods of setting PID controllers

Methods of setting PID controllers

According to standard literature a transfer PID (ideal) is:

$$C(s) = r_o + \frac{r_{o1}}{s} + r_1 s = r_o\left(1 + \frac{1}{T_I s} + T_D s\right)$$

Remark: - realistic

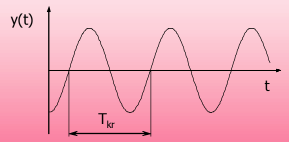
$$C(s) = r_o\left(1 + \frac{1}{T_I s} + \frac{T_D s}{\frac{T_D}{N}s + 1}\right) \qquad N \in <3;1000>$$

6.Methods of setting PID controllers

Classic methods of setting PID controllers

A) Ziegler-Nichols – from critical gain: Controller ⇒ r_{kr}

Vibration period – T_{kr}



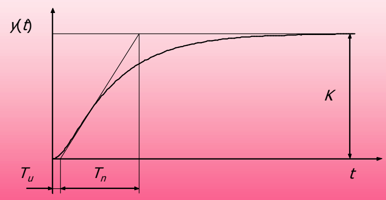
Controller tuning

	r_o	T_I	T_D
P	0,5 k_{rk}	-	-
PI	0,45 k_{rk}	0,85 T_{kr}	-
PID	0,6 k_{rk}	0,5 T_{kr}	0,125 T_{kr}

6.Methods of setting PID controllers

Remark: system of 1st and 2nd order of P controller isn't possible quiver

B) From unit step response (aperiodic type)



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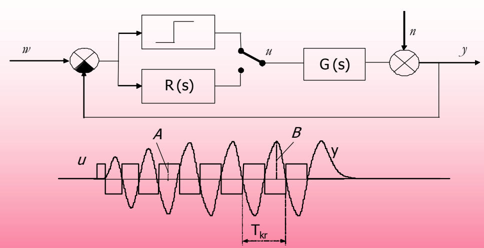
	r_o	T_I	T_D
P	$\frac{1}{a}$	-	-
PI	$0,9\frac{1}{a}$	3L	-
PID	$1,2\frac{1}{a}$	2L	0,5L

C) By the help relay – Hägglund (1983) – autotuning

Values of ultimate gain and critical vibration period are possible determine also other way. Insertion relay to feedback of control loop and calculation according to figure.

6.Methods of setting PID controllers



$$r_{rk} = \frac{4}{\pi}\frac{A}{B}$$

A … high of relay

B … vibration amplitude

6.Methods of setting PID controllers

D) Balanced setting (Klan, Gorez 1996-2001)

Control transfer $G(s)=\dfrac{b_0}{a_n s^n+\ldots+a_1 s+a_0}$

PI controller r_0 = 0,5 if $\dfrac{b_0}{a_0}=1$

TI = 0,5 T_{kr}

Or r0 = 0,5 a_0/b_0 if $\dfrac{b_0}{a_0}\neq 1$

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6.Methods of setting PID controllers

E) Usage of critical stability for controllers design

Principle: $G(s)=\dfrac{b(s)}{a(s)}$ controlled system

$C(s)=\dfrac{q(s)}{p(s)}$ controller

Process: 1) Calculate characteristic polynomial *ap + bq*

2) Apply stability criterion – usually Routh-Schur

3) Make inequation system

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6.Methods of setting PID controllers

F) Cohen – Coon method

from transfer of three-parameters model $G(s)=\dfrac{K}{Ts+1}e^{-\Theta s}$

This method is proposed so that gives damping ratio ¼. It means that proposed controller will be provides control process, whose second vibration response will have quarter of first amplitude.

Parameters of controller for Cohen-Coon method

	k_r	T_I	T_D
P	$\dfrac{1}{K.r}\left(1+\dfrac{r}{3}\right)$	-	-
PI	$\dfrac{1}{K.r}\left(0,9+\dfrac{r}{12}\right)$	$\dfrac{30+3r}{9+20r}\Theta$	-
PID	$\dfrac{1}{K.r}\left(\dfrac{4}{3}+\dfrac{r}{4}\right)$	$\dfrac{32+6r}{13+8r}\Theta$	$\dfrac{4}{11+2r}\Theta$

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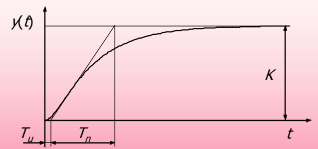
6.Methods of setting PID controllers

For parameters *r* is valid $r=\dfrac{\Theta}{T}$



G) Whiteley's standard forms

1) Tables are for transfer $K_{W/Y}$

$$K_{w,y}=\frac{bq}{ap+bq}=\frac{G.C}{1+G.C}$$

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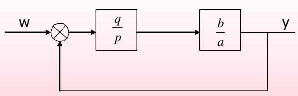
6.Methods of setting PID controllers



Example 1: $G(s)=\dfrac{1}{s^2+3s+2}$ (cannot be set Z-N)

$C(s)=\dfrac{q_2 s^2+q_1 s+q_0}{s}$ (ideal PID)

$$K_{W/Y}=\frac{q_2 s^2+q_1 s+q_0}{s^2+3s+2s+q_2 s^2+q_1 s+q_0}$$

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6.Methods of setting PID controllers

Using Whiteley 3 for n=3

$s^0:\quad q_0=1$

$s^1:\quad 2+q_1=6,7\Rightarrow q_1=4,7$

$s^2:\quad 3+q_2=6,7\Rightarrow q_2=3,7$

2) Usually is necessary make transformation so that marginal coefficients were equal to 1

$$S=\left(\frac{a_0}{a_n}\right)^{\frac{1}{n}}q$$

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6.Methods of setting PID controllers

Example 2: $G(s)\frac{2}{s(s+10)^2}$ $C=\frac{q_1 s+q_0}{s}$

$$K_{w,y}=\frac{2q_1 s+2q_0}{s^4+20s^3+100s^2+2q_1 s+2q_0}=\frac{\frac{2q_1}{2q_0}s+1}{\frac{1}{2q_0}s^4+\frac{20}{2q_0}s^3+\frac{100}{2q_0}s^2+\frac{2q_1}{2q_0}s+1}$$

$$S=\left(\frac{a_0}{a_4}\right)^{\frac{1}{4}}q=(2q_0)^{\frac{1}{4}}q$$

Denominator of transfer after substitution

$$\frac{1}{2q_0}(2q_0)^{\frac{1}{4}}q^4+\frac{20}{2q_0}(2q_0)^{\frac{3}{4}}q^3+\frac{100}{2q_0}(2q_0)^{\frac{2}{4}}+\frac{2q_1}{2q_0}(2q_0)^{\frac{1}{4}}s+1$$

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6.Methods of setting PID controllers

Using Whiteley 2 n=4 1 7,2 16 12 1

q^4: $1=1$

q^3: $\frac{20}{2q_0}(2q_0)^{\frac{3}{4}}=7{,}2$

q^2: $\frac{100}{2q_0}(2q_0)^{\frac{1}{2}}=16$

q^1: $\frac{2q_1}{2q_0}(2q_0)^{\frac{1}{4}}=12$

q^0: $1=1$

Solution: $q_0=29{,}768$, $q_1=128{,}6$

Other methods

Minimum of conicoid

Optimal module

Half damping

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6.Methods of setting PID controllers

Saturation of controller – Wind up

Way of limitation $u\in<u_{\min};u_{\max}>$

In integral component, that integrate control error and increase actuating value. Occurs saturation and controller begins behave as a two step controller.

Elimination:

1) Limitation on change of desired variable
2) Turn of integration conditionally
3) Back calculation – Antiwind up

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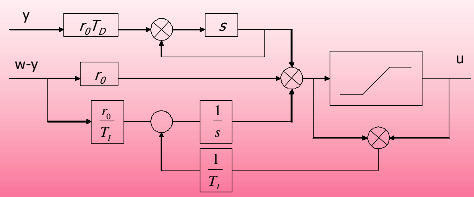
6.Methods of setting PID controllers

PID – anti wind up



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6.Methods of setting PID controllers

Recommended form $T_t=\sqrt{T_I T_D}$

Modification PID $u(t)=r_0\left[\beta w-y+\frac{1}{T_I}\int(w-y)\,d\tau-T_D\frac{dyf}{dt}\right]$

$\tau y_f'+y_f=y$

0 < ß ≤ 1 lowers overshoots

$$u(t)=r_0 e(t)+\frac{r_0}{T_I}\int e(\tau)\,d\tau+r_0 T_D e'(t)$$

$$u'(t)=r_0 e'(t)+\frac{r_0}{T_I}e(t)+r_0 T_D e''(t)$$

Approximation of formula $x'(t_k)=\frac{x(t_k)-x(t_{k-1})}{\Delta}$

$x''(t_l)=\frac{x(t_l)-2x(t_{l-1})+x(t_{l-2})}{\Delta^2}$? - period

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6.Methods of setting PID controllers

Substitution to basic control: $\frac{u_k-u_{k-1}}{\Delta}=r_0\frac{e_k-e_{k-1}}{\Delta}+\frac{r_0}{T_I}e_k+r_0 T_D\frac{e_k+e_{k-1}+e_{k-2}}{\Delta^2}$

after modification $u_k=u_{k-1}+c_0 e_k+c_1 e_{k-1}+c_2 e_{k-2}$

$$c_0=r_0\left(1+\frac{\Delta}{T_I}+\frac{T_D}{\Delta}\right)$$

$$c_1=-r_0\left(1+2\frac{T_D}{\Delta}\right)$$

$$c_2=\frac{r_0 T_D}{\Delta}$$

Remark:

1) Applies for small ?

2) Exist a lot of approximations

3) Definition $qx_k=x_{k-1}\Rightarrow\frac{u_k}{e_k}=\frac{c_0+c_1 q+c_2 q^2}{1-q}$

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6.Methods of setting PID controllers

Algebraic methods of setting controllers

Rings and fields

Ring O is non-empty set, for her elements (a, b, c ∈ O) are defined operations addition and multiplication (commutative field), and are fulfilled following axioms

I.
$a+b\in\Omega$
$a+b=b+a$ (for commutative ring)
$\exists\Theta\in\Omega\ \ a+\Theta=\Theta+a=a$ (exist zero element)
$\forall a\in\Omega\ \ \exists(-a)\in\Omega\ \ a+(-a)=\Theta$

II.
$a\cdot b\in\Omega$
$a\cdot b=b\cdot a$ (commutative law)
$\exists e\in\Omega\ \ e\cdot a=a\cdot e=a$ (exist unit element)

III.
$(a+b)\cdot c=a\cdot c+b\cdot c$ (distributive law)
$(a+b)+c=a+(b+c)$ (associative law)
$(a\cdot b)\cdot c=a\cdot(b\cdot c)$ (associative law)

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6.Methods of setting PID controllers

$\forall a\neq\Theta\in\Omega\ \ \exists\left(a^{-1}\right)\ \ a\cdot a^{-1}=e$ dividing axiom

If isn't fulfilled axiom II.d (axiom of dividing) ⇒ set M is called commutative ring

Examples:

integral numbers, polynomial, sets R_{ps} – rings

rational numbers, complex numbers, transfers – fields

Remark:

1) Also exist noncommutative rings and fields

2) Ring is algebraic group in face of addition and half group in face of multiplication

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8.7. Syntéza regulátorů

7.Synthesis of controllers

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7.Synthesis of controllers

Diophantine equations – one equation in two unknown

$ax+by=c$ in commutative ring

Applies:

1) Diophantine equation have solution ⇔ least common divider *(a, b)* divide *c*

2) If diophantine equation have solution ⇒ exists infinite solutions

$$x = x_0 + \frac{b}{hcd(a,b)}t$$

h.c.d = highest common divisor

$$y = y_0 - \frac{a}{hcd(a,b)}t$$

t ... is any element of ring

7.Synthesis of controllers

Example:

4x + 6y = 10 4x + 6y = 9 don't have solution

x = 1 + 3t *t* ... is integer number

y = 1 – 2t

Important concept in ring is a divisibility. It is possible write:

1) *a* divide *b* ⇔ ∃ *d* ∈O *b=a.d*

2) *d* is divider *a* and *b* ⇔ $a = a_0 d$ $b=b_0 d$

3) *d* is h.c.d *(a, b)* ⇔ d is divisible all dividers a, b

4) element x∈O exists inverse again in O to him, is called unit (invertible)

5) short divide $a/b = p$ zb. d ⇔ $a = bp + d$

long divide $a/b \in$ in superior field

7.Synthesis of controllers

Example:

1. Ring of integer numbers – divisibility by prime factor

Units: +1, -1 unit element: +1

2 individe 5 ; 6 divide 30

5 / 2 = 2 remainder 1 short divide

5 / 2 ? rational

h.c.d (150, 63): 150 : 63 = 2 remainder 24 Euclid's algorithm

63 : 24 = 2 remainder 15

24 : 15 = 1 remainder 9

15 : 9 = 1 remainder 6

9 : 6 = 1 remainder 3

6 : 3 = 2 remainder 0

7.Synthesis of controllers

Generalized Euclid's algorithm

$$\left(\begin{array}{cc|c}1 & 0 & 150\\ 0 & 1 & 63\end{array}\right) \approx \left(\begin{array}{cc|c}1 & -2 & 24\\ 0 & 1 & 63\end{array}\right) \approx \left(\begin{array}{cc|c}1 & -2 & 24\\ -2 & 5 & 15\end{array}\right) \approx \left(\begin{array}{cc|c}3 & -7 & 9\\ -2 & 5 & 15\end{array}\right) \approx$$

$$\approx \left(\begin{array}{cc|c}3 & -7 & 9\\ 5 & 12 & 6\end{array}\right) \approx \left(\begin{array}{cc|c}8 & -19 & 3\\ -5 & 12 & 6\end{array}\right) \approx \left(\begin{array}{cc|c}8 & -19 & 3\\ -21 & 50 & 0\end{array}\right) \Rightarrow \begin{array}{l} ax+by=hcd(a,b)\\ 150x8+63x(-19)=3\\ +(-21)x150+50x63=0\end{array}$$

2.Ring of polynomial

divisibility ≡ by means of roots

units ≡ nonzero constants

Example: $a(s) = s^2+3s+2 = (s+1)(s+2)$

$b(s) = s^2+1{,}5s+0{,}5 = (s+1)(s+0{,}5)$

7.Synthesis of controllers

Generalization of Euclid's algorithm

$$-1\left(\begin{array}{cc|c}1 & 0 & s^2+3s+2\\ 0 & 1 & s^2+1{,}5s+0{,}5\end{array}\right)_{\frac{1}{1,5}} \approx \left(\begin{array}{cc|c}1 & 0 & s^2+3s+2\\ -1 & 1 & -1{,}5s-1{,}5\end{array}\right) \approx \left(\begin{array}{cc|c}1 & 0 & s^2+3s+2\\ \frac{2}{3} & -\frac{2}{3} & s+1\end{array}\right) \approx -s$$

$$\left(\begin{array}{cc|c}1+\frac{2}{3}s & -\frac{2}{3} & 2s+2\\ \frac{2}{3} & -\frac{2}{3} & s+1\end{array}\right) -2 \approx \left(\begin{array}{cc|c}-\frac{1}{3}+\frac{2}{3}s & -\frac{2}{3}s+\frac{4}{3} & 0\\ \frac{2}{3} & -\frac{2}{3} & s+1\end{array}\right) \Rightarrow hcd(a,b) = s+1$$

3. Ring R_{ps}: proper and stable rational function

Example: $\frac{1}{s+2}$ $\frac{-s+1}{s^2+3s+2}$ $\frac{+s}{s+1}$ $\in R_{PS}$

s $\frac{1}{s}$ $\frac{s^2+1}{s+3}$ $\frac{3}{s-1}$ $\notin R_{PS}$

7.Synthesis of controllers

Divisibility: by means of unstable zeros (including 8)

Units (invertible): stable numerator and denominator of real order 0.

⇒ using for robust control, H_8, ...

4. Also exists noncommutative rings – MIMO systems described by the help matrixes

Diophantine equations in ring of polynomial

$ax+by=c$ supposition: comprime a, b ⇒ exists solution

Method of undetermined coefficients

Searches particular solution, so that compares coefficients on left side and right side of equation ⇒ transmission to system of algebraic equations.

estimation of orders:
or. x = or. b – 1, or. y = or. a – 1 } If or. a + or. b > or. c
or. x = or. c - or. a, or. y = or. a – 1 } If or. a + or. b ≤ or. c

7.Synthesis of controllers

Polynomial one-dimensional synthesis

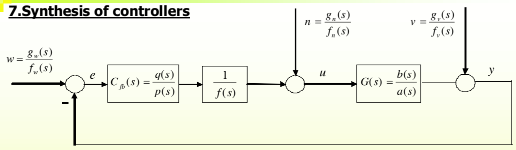
Main Aims:

1. determine system stability
2. determine asymptotic tracking, $y(t)$ ⇒ w
3. determine proper transfers of controller (no derivative)
4. Put poles to defined roots (poles of characteristic equation)

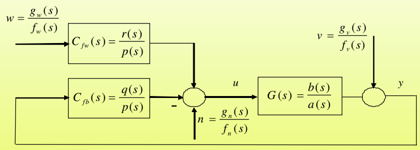
1DOF (FB) – system with one-degree of freedom (only with feedback part of controller)

2DOF (FBFW) – system with two-degree of freedom (with feedback and feedforward part of controller)

7.Synthesis of controllers



system with one-degree of freedom (1DOF)



system with two-degree of freedom (2DOF)

7.Synthesis of controllers

Algebra (without 1/f)

$$y=\frac{b}{a}\frac{q}{p}(w-y) \qquad y=\frac{b}{a}\left(\frac{r}{p}w-\frac{q}{p}y\right)$$

$$y=\frac{bq}{ap+bq}w \qquad y=\frac{br}{ap+bq}w$$

$$e=\left(1-\frac{bq}{ap+bq}\right)\frac{q}{f}=\frac{ap}{ap+bq}\frac{q}{f} \qquad e=\left(1-\frac{br}{ap+bq}\right)\frac{q}{f}$$

Achieving of aim

e → 0 ⇔ f divide ap

afp + bq = m or. m ≥ 2 or. a	ap + bq = m ft + br = m or. m ≥ 2 or. a – 1

7.Synthesis of controllers

Selection $m(s)$ – stable: $m(s)=(s+m_0)^n$ n...is degree of polynomial $a(s)$ [order]

$m_0>0$

Polynomial synthesis of 1st order $G(s)=\frac{b_0}{s+a_0}$

a) 1DOF $(s+a_0)sp_0+b_0(q_0+q_1s)=(s+m_0)^2$

$$\left.\begin{aligned} s^2:&\quad p_0=1\\ s^1:&\quad a_0p_0+b_0q_1=2m_0\\ s^0:&\quad b_0q_0=m_0^2\end{aligned}\right\} \quad p_0=1,\quad q_1=\frac{2m_0-a_0}{b_0},\quad q_0=\frac{m_0^2}{b_0}$$

Basic control:

$$sU=(q_0+q_1s)[W-Y]$$
$$u'(t)=q_0(w-y(t))+q_1(w'-y(t))$$
$$u(t)=q_0\int(w-y(\tau))d\tau+q_1(w-y(t))$$

PI controller with transfer function

$$C(s)=\frac{q(s)}{p(s)}=\frac{q_1s+q_0}{s}$$

$m_0>0$ is tuning element

7.Synthesis of controllers

b) 2DOF $(s+a_0)p_0+b_0q_0=s+m_0$ $\qquad st_0+b_0r_0=s+m_0$

$$\begin{aligned} s^1:&\quad p_0=1 & s^1:&\quad t_0=1\\ s^0:&\quad a_0p_0+b_0q_0=m_0 & s^0:&\quad b_0r_0=m_0\end{aligned}$$

$$p_0=1,\quad q_0=\frac{m_0-a_0}{b_0} \qquad t_0=1,\quad r_0=\frac{m_0}{b_0}$$

Basic control:

$$u(t)=r_0w(t)-q_0y(t)$$

$$\left[u=\frac{m_0}{b_0}w-\frac{m_0-a_0}{b_0}y\rightarrow\beta w-y\right]$$ PP controller

7.Synthesis of controllers

Remark:

1) What if is $w(t)$ for example harmonic signal?

2) What if in system other disturbances?

Fw Fv Fd have to divide P (denominator of controller)

3) Generally

$$m(s) = s^2 + m_1 s + m_0$$

$$m_1 > 0 \qquad m_0 > 0$$

7.Synthesis of controllers

Polynomial synthesis for system of 2nd order

Consider plant of 2nd order with transport function

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

Propose controller for asymptotic tracking of desired value for 1DOF a 2DOF configuration.

Control system 1DOF

Basic diophantine equation

$$\left(s^3 + a_1 s^2 + a_0 s\right)\left(p_1 s + p_0\right) + \left(b_1 s + b_0\right)\left(q_2 s^2 + q_1 s + q_0\right) = \left(s + m\right)^4$$

7.Synthesis of controllers

After modification:

$$\begin{array}{llllll}
s^4: & p_1 & & & & = 1 \\
s^3: & a_1 p_1 + & p_0 + & b_1 q_2 & & = 4m \\
s^2: & a_0 p_1 + & a_1 p_0 + & b_0 q_2 + & b_1 q_1 & = 6m^2 \\
s^1: & & a_0 p_0 + & & b_0 q_1 + \; b_1 q_0 & = 4m^3 \\
s^0: & & & & b_0 q_0 & = m^4
\end{array}$$

After resolution of equation system, we get coefficients for final controller

$$C(s) = \frac{q_2 s^2 + q_1 s + q_0}{s\left(p_1 s + p_0\right)}$$

Control system 2DOF

Determine degree of polynomials as 1DOF, and make pair of diophantine equations, that is,

$$\left(s^2 + a_1 s + a_0\right)\left(p_1 s + p_0\right) + \left(b_1 s + b_0\right)\left(q_1 s + q_0\right) = \left(s + m\right)^3$$

$$s\left(t_2 s^2 + t_1 s + t_0\right) + \left(b_1 s + b_0\right) r_0 = \left(s + m\right)^3$$

7.Synthesis of controllers

For first equation: After comparing we get system of 4 equations

$$\begin{array}{llllll}
s^3: & p_1 & & & & = 1 \\
s^2: & a_1 p_1 + & p_0 + & b_1 q_1 & & = 3m \\
s^1: & a_0 p_1 + & a_1 p_0 + & b_0 q_1 + & b_1 q_0 & = 3m^2 \\
s^0: & & a_0 p_0 + & & b_0 q_0 & = m^3
\end{array}$$

For second equation

$$\begin{array}{llllll}
s^3: & t_2 & & & & = 1 \\
s^2: & & t_1 & & & = 3m \\
s^1: & & & t_0 + & b_1 r_0 & = 3m^2 \\
s^0: & & & & b_0 r_0 & = m^3
\end{array}$$

Basic control

$$p_1 u' + p_0 = r_0 w - q_1 y' - q_0 y$$

$$u = \frac{1}{p_1}\left\{-p_0 \int u + r_0 \int w - q_1 y - q_0 \int y\right\}$$

7.Synthesis of controllers

Remark:

Better is solution of 2nd equation $r = r_0 + q_0 s$, than controller have form of a Aström‘s controller

$$u = \frac{1}{p_1}\left\{-p_0 \int u - r_0 \int w - q_0 \int \left(w - y\right) - q_1 y\right\}$$

Solution is given by final controller $C_{FB}(s) = \dfrac{q_1 s + q_0}{p_1 s + p_0}$

In terms of regulation is important only one parameters (r_0) and from last equation results for him:

$$r_0 = \frac{m^3}{b_0}$$

7.Synthesis of controllers

Suggestion of Robustness

nominal: $G = \dfrac{b}{a} \xleftrightarrow{stab.} C = \dfrac{q}{p}$

perturbative: $\tilde{G} = \dfrac{\tilde{b}}{\tilde{a}} \longrightarrow \tilde{C} = \dfrac{\tilde{q}}{\tilde{p}}$

Will be combination $C + \tilde{G}$ or $\tilde{C} + \tilde{G}$ make stable closed-loop control?

Example: $G = \dfrac{5}{s-1}$ $\qquad C = \dfrac{q_1 s + q_0}{s}$

$\tilde{G} = \dfrac{5}{s-2}$ $\qquad ap + bq = s^2 + \left(5q_1 - 1\right)s + 5q_0$

7.Synthesis of controllers

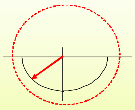
a) $q_0 = 1$ $q_1 = 0{,}3$ is stable for *G*, but no for $\tilde{G}$

b) $q_1 = 1$ $q_1 = 1$ is stable for *G* and also for $\tilde{G}$ $\quad \|G(s)\| = \sup_{\varpi \in \mathrm{Re}} |G(j\varpi)|$

Instruments for study robustness are more complicated

a) Norm H_8 in space R_{ps}

b) Interval polynomials,



Interval polynomials $\left\{ \begin{array}{ll} a(s) = s^2 + a_1 s + 3 & a_1 \in \langle 1; 2 \rangle \\ p(s) = s^2 + p_0 s & p_0 \in \langle 0{,}5; 1 \rangle \end{array} \right.$

$$a(s)\,p(s) = s^4 + (p_0 + a_1)s^3 + (a_1 p_0 + 3)s^2 + 3p_0 s$$

Notions: Kharitonov's theorem

Theorem about edges

7.Synthesis of controllers

Algebraic solution for systems with transport delay $\quad G(s) = \dfrac{b(s)}{a(s)} e^{-Ls}$

$$L \equiv \Theta \equiv \tau \equiv d \equiv T_d$$

Basic ways of approximation of transport delay are:

- Neglect of transport delay $\quad e^{-Ls} \approx 1$
- Taylor's expansion (1st order) in numerator $\quad e^{-\Theta s} \approx 1 - Ls$
- Taylor's expansion (1st order) in denominator $\quad e^{-Ls} \approx \dfrac{1}{e^{-Ls}} \approx \dfrac{1}{1 + Ls}$
- Padé approximation $\quad e^{-Ls} = \dfrac{e^{-\frac{L}{2}s}}{e^{\frac{L}{2}s}} \approx \dfrac{1 - \frac{L}{2}s}{1 + \frac{L}{2}s}$

7.Synthesis of controllers

Taylor's expansion – generally $\quad e^{Ls} = \sum_{k=0}^{\infty} \dfrac{1}{k!}(Ls)^k$

Example: $\quad G(s) = \dfrac{3}{s+1} e^{-2s} \approx \dfrac{3}{s+1}$

$$\approx \frac{3 - 2s}{s+1}$$

$$\approx \frac{3}{(s+1)(s+2)}$$

$$\approx \frac{3(s+1)}{(s+1)(s+2)}$$

7.Synthesis of controllers

Multidimensional systems

Remark:

For marking of multidimensional systems is used shortcut „MIMO" (Multi-Input, Multi output), for specific case of system with two inputs and two outputs „TITO" (Two-Input Two-Output) and for systems one-dimensional „SISO" (Single-Input Single-Output). Generalization of description through one differential equation is description of systems by the help differential equations system. For case with two input, two output and dynamic of 1st order is it:

$$y_1'(t) + a_1 \cdot y_1(t) + a_2 \cdot y_2(t) = b_1 \cdot u_1(t) + b_2 \cdot u_2(t)$$
$$a_3 \cdot y_1(t) + y_2'(t) + a_4 \cdot y_2(t) = b_3 \cdot u_1(t) + b_4 \cdot u_2(t)$$

After using Laplace transform (with zero initial conditions)

$$(s + a_1) \cdot Y_1(s) + a_2 \cdot Y_2(s) = b_1 \cdot U_1(s) + b_2 \cdot U_2(s)$$
$$a_3 \cdot Y_1(s) + (s + a_4) \cdot Y_2(s) = b_3 \cdot U_1(s) + b_4 \cdot U_2(s)$$

7.Synthesis of controllers

After transcript to matrix form is possible this equation write like this:

$$\begin{pmatrix} s + a_1 & a_2 \\ a_3 & s + a_4 \end{pmatrix} \begin{pmatrix} Y_1(s) \\ Y_2(s) \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix}$$

Applies: $\quad \mathbf{A}(s)\mathbf{Y}(s) = \mathbf{B}(s)\mathbf{U}(s)$

Where: matrix A have size *l* x *l* and matrix B *l* x *m*

Matrix transfer of multidimensional systems is matrix of rational fractional function

$$\mathbf{G}(s) = \mathbf{A}^{-1}(s)\mathbf{B}(s)$$

7.Synthesis of controllers

Applies:

1) To each of left matrix fraction exists also right $\quad A^{-1}B = B_p A_p^{-1}$

a) Dimensions *B*, B_p are equal (*l* x *m*), dimensions *A*, A_p are unequal, *A* (*l* x *l*) and A_p (*m* x *n*)

b) $\det A \approx \det A_p$ (same roots)

2) Stability: MIMO is stable ⇔ $\det A \approx \det A_p$ is stable

3) MIMO is proper ⇔ if all transfers in *G* is proper

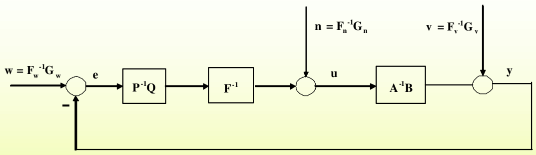
Example:

$$A^{-1}B = \frac{1}{s^2 + (a_1 + a_2)s + a_1 a_4 - a_2 a_3} \begin{pmatrix} s + a_4 & -a_2 \\ -a_3 & s + a_1 \end{pmatrix} \begin{pmatrix} b_1 & b_4 \\ b_3 & b_4 \end{pmatrix}$$

Transfer matrix have form: $\quad \mathbf{G}(s) = \begin{pmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{l1}(s) & G_{l2}(s) & \cdots & G_{lm}(s) \end{pmatrix}$

7.Synthesis of controllers

Synthesis of multidimensional control system



Applies: $A^{-1}B = B_P A_P^{-1}$ is possible turn $P^{-1}Q = Q_P P_P^{-1}$

From basic block diagram algebra is given following relationship:

$$\mathbf{Y} = \mathbf{A}^{-1}\mathbf{B}\mathbf{P}^{-1}\mathbf{Q}(\mathbf{W} - \mathbf{Y})$$

$$(\mathbf{I} + \mathbf{A}^{-1}\mathbf{B}\mathbf{P}^{-1}\mathbf{Q})\mathbf{Y} = \mathbf{A}^{-1}\mathbf{B}\mathbf{P}^{-1}\mathbf{Q}\mathbf{W}$$

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7.Synthesis of controllers

After multiplication with matrix A $\left(\mathbf{A} + \mathbf{B}\mathbf{Q}_P\mathbf{P}_P^{-1}\right)\mathbf{Y} = \mathbf{B}\mathbf{P}^{-1}\mathbf{Q}\mathbf{W}$

and after next modifications $\left(\mathbf{A}\mathbf{P}_P + \mathbf{B}\mathbf{Q}_P\right)\mathbf{P}_P^{-1}\mathbf{Y} = \mathbf{B}\mathbf{P}^{-1}\mathbf{Q}\mathbf{W}$

applies identity $\mathbf{Y} = \mathbf{P}_P\left(\mathbf{A}\mathbf{P}_P + \mathbf{B}\mathbf{Q}_P\right)^{-1}\mathbf{B}\mathbf{P}^{-1}\mathbf{Q}\mathbf{W}$

$$\mathbf{P}_P\left(\mathbf{A}\mathbf{P}_P + \mathbf{B}\mathbf{Q}_P\right)^{-1}\mathbf{B} = \mathbf{B}_P\left(\mathbf{P}\mathbf{A}_P + \mathbf{Q}\mathbf{B}_P\right)^{-1}\mathbf{P}$$

is possible write $\mathbf{Y} = \underbrace{\mathbf{B}_P\left(\mathbf{P}\mathbf{A}_P + \mathbf{Q}\mathbf{B}_P\right)^{-1}\mathbf{Q}}_{\mathbf{K}_{w,y}}\mathbf{W}$

Matrix $K_{W/Y}$ define transfer of control

Synthesis of MIMO controller: $\mathbf{P}\mathbf{A}_P + \mathbf{Q}\mathbf{B}_P \approx \mathbf{A}\mathbf{P}_P + \mathbf{B}\mathbf{Q}_P$ is stable

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7.Synthesis of controllers

For formulation condition of asymptotic tracking desired value is expressed control error

$$\mathbf{E} = \mathbf{W} - \mathbf{Y} = \left[\mathbf{I} - \mathbf{B}_P\Big(\underbrace{\mathbf{P}\mathbf{A}_P + \mathbf{Q}\mathbf{B}_P}_{\mathbf{D}}\Big)^{-1}\mathbf{Q}\right]\mathbf{W}$$

$$\mathbf{E} = \left[\mathbf{I} - \mathbf{B}_P\mathbf{D}^{-1}\mathbf{Q}\right]\mathbf{F}_w^{-1}\mathbf{G}_w$$

Using analogue analysis as for one-dimensional system is possible to claim, that for asymptotic tracking have to be reduced element F_w included in W. It is possible by using comparator.

That is appeared in controller, and his form is: $\mathbf{F} = \mathbf{F}_w$

$$\mathbf{C} = \mathbf{Q}_P\left(\mathbf{P}_P\mathbf{F}\right)^{-1}$$

Definition: MIMO system is invariant (disturbance rejection) ⇔ compensate on out ⇒ result: denominator Fv have to be in denominator of controller

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7.Synthesis of controllers

Matrix diophantine equation *ax + by = c*

(left) $\mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{Y} = \mathbf{C}$

(bilateral) $\mathbf{A}\mathbf{X} + \mathbf{Y}\mathbf{B} = \mathbf{C}$

(right) $\mathbf{X}\mathbf{A} + \mathbf{Y}\mathbf{B} = \mathbf{C}$

Bilateral equation is not solvable. Equation can be solved by multiplication term by term

$$\left(\begin{array}{c|c} A & B \\ I & 0 \\ 0 & I \end{array}\right) \xrightarrow[\text{modifications}]{\text{Elementary column}} \left(\begin{array}{c|c} C & 0 \\ X_0 & Z_1 \\ Y_0 & Z_2 \end{array}\right)$$

$$AX_0 + BY_0 = C$$

$\left.\begin{array}{l} Z_1 = -B_p \\ Z_2 = A_p \end{array}\right\}$ turned matrix fraction

$$\left(\begin{array}{ccc} A & I & 0 \\ \hline B & 0 & I \end{array}\right) \xrightarrow[\text{modifications}]{\text{Elementary row}} \left(\begin{array}{ccc} C & X_1 & Y_1 \\ \hline 0 & Z_1 & Z_2 \end{array}\right)$$

$$X_1A + Y_1B = C$$

$\left.\begin{array}{l} Z_1 = -B_p \\ Z_2 = A_p \end{array}\right\}$ turned matrix fraction

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7.Synthesis of controllers

Conversion to scalar polynomial equation by multiplication term by term

Example: two input and two output (2x2)

$$A = \begin{pmatrix} s+2 & 3 \\ 2 & s+0{,}5 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0{,}5 \\ 0 & 2 \end{pmatrix} \quad B = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} \quad M = \begin{pmatrix} (s+1)^2 & 0 \\ 0 & (s+1)^2 \end{pmatrix}$$

-0,5

$$\left(\begin{array}{cc|cc} s^2+2s & 3s & 1 & 0{,}5 \\ 2s & s^2+0{,}5s & 0 & 2 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \approx \left(\begin{array}{cc|cc} (s+1)^2 & 0 & 1 & 0 \\ 2s & s^2+0{,}5s & 0 & 2 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & -3s & 1 & -0{,}5 \\ 0 & 0 & 0 & 1 \end{array}\right) \approx \left(\begin{array}{cc|cc} (s+1)^2 & 0 & 1 & 0 \\ 0 & (s+1)^2 & 0 & 2 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline s+1 & 3{,}375s-\frac{1}{4} & 1 & -0{,}5 \\ -2s & 0{,}75s+\frac{1}{2} & 0 & 1 \end{array}\right)$$

-3s

0,75s + 0,5

+1

s

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7.Synthesis of controllers

Result: $P_P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad Q_P = \begin{pmatrix} s+1 & -3{,}375s - 0{,}25 \\ -2s & 0{,}75s + 0{,}5 \end{pmatrix}$

$$F_U = Q(w - y)$$

$$\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} s+1 & -3{,}375s - 0{,}25 \\ -2s & 0{,}75s + 0{,}5 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

$\left.\begin{array}{l} u_1 = e_1 + \int e_1 - 3{,}375e_2 - 0{,}25\int e_2 \\ u_2 = -2e_1 + 0{,}75e_2 + 0{,}5\int e_2 \end{array}\right\}$ Generalized MIMO PI controller

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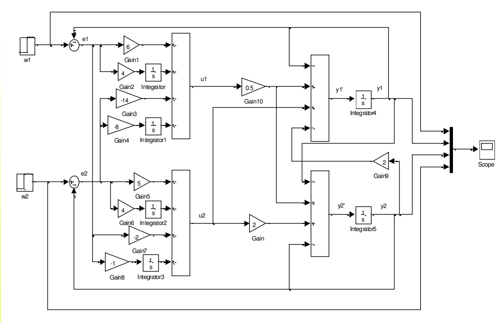
7.Synthesis of controllers



Schema of two-dimensional control system

7.Synthesis of controllers

Plant:

$$y' = -2y_1 - 3y_2 + u_1 + 0,5u_2$$
$$y_2' = -2y_1 - 0,5y_2 + 2u_2$$

Controller:

$$u_1 = e_1 + \int e_1 - 3,375e_2 - 0,25\int e_2$$
$$u_2 = -2e_1 + 0,75e_2 + 0,5\int e_2$$

9 VZOROVÉ PROTOKOLY

Jedním ze zadaných úkolů bylo vytvořit vzorové protokoly, které by měli sloužit pro podporu seminárních cvičení. Níže je uveden popis jednotlivých protokolů.

První protokol

První protokol je zaměřen na vnější popis a na analýzu spojitého dynamického systému. Každý student obdrží své individuální zadání koeficientů a_2, a_1, a_0, b_0, které dosadí do zadané diferenciální rovnice. A následně vypracuje zadané úkoly,tj. např. určení pólů, nul, stability, přechodové, impulsní charakteristiky atd.

Druhý protokol

Druhý protokol se zabývá syntézou regulačního obvodu. Student nejprve navrhne spojitý regulátor pomocí kritéria stability a taky některou z vybraných klasických metod návrhů parametrů regulátoru. Dále se navrhne regulátor pomocí polynomiální syntézy 1DOF a 2DOF konfigurace. Také se k přenosu přidá dopravní zpoždění a simuluje se průběh s použitím Smithova predikátoru a bez jeho použití. Nakonec se přidané dopravní zpoždění aproximuje a navrhnou se parametry regulátoru jednou z polynomiálních syntéz 1DOF nebo 2DOF.

Třetí protokol

V třetím protokolu je úkolem určit stavový popis lineárních spojitých dynamických systémů. Vyřeší se matice řiditelnosti a pozorovatelnosti a na jejich základě se určí jestli je systém řiditelný a pozorovatelný.

9.1 První protokol – vnější popis a analýza LSDS

TOMAS BATA UNIVERSITY IN ZLIN Faculty of Applied informatics			
Name:		**Grade:**	II
Subject:	Automatic control theory	**Group:**	
Theme:	Outer description and analysis continuous dynamic system		

SISO (single input-output) linear continuous dynamic system is given by differential equation

$$a_2 y''(t) + a_1 y'(t) + a_0 y(t) = b_0 u(t)$$

Tasks:

1) Write transfer function, consider zero initial conditions.

2) Determine zeros, poles, and relative order.

3) Decide about stability, periodicity, phase characteristic

4) Figure out unit function response and depicture unit step response.

5) Figure out impulse function and depicture impulse response.

6) Determine amplitude-phase frequency response, depicture amplitude-phase frequency response in complex plane (Nyquist diagram) and depicture frequency response in logarithmic coordinate (Bode diagram).

Elaboration

1)

Settings value: $a_2 = 1$, $a_1 = 5$, $a_0 = 4$, $b_0 = 2$

Differential equation: $y''(t) + 5y'(t) + 4y(t) = 2u(t)$

Transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{2}{s^2 + 5s + 4} = \frac{2}{(s+1)(s+4)} = \frac{(s - n_1)}{(s - p_1)\cdot(s - p_2)}$$

2)

Zeros (roots of numerator): ∞**,** ∞

Poles (roots of denominator): **-1, -4**

Order (degree of denominator): **2**

Relative order (degree of denominator minus degree of numerator): **2-0 = 2**

3)

Stability: system is **stable** (all of the poles lies in left part of complex plane)

Periodicity: system is **aperiodic** (all of the poles lies on real axis)

system is **minimum phase** (all of the zeros are in infinite)

4)

Unit step response

Step function is response to unit step function at zero initial conditions

$$h(t) = L^{-1}\{H(s)\} = L^{-1}\left\{\frac{G(s)}{s}\right\} = L^{-1}\left\{\frac{2}{s(s^2 + 5s + 4)}\right\}$$

Calculation by the help of residues:

$$f(t) = \sum_{s=s_i} res\left[F(s)e^{st}\right] = \lim_{s \to s_i}\left[(s - s_i)F(s)e^{st}\right]$$

$$\text{f(t)} = \lim_{s\to 0}\frac{2}{(s+1)(s+4)}e^{st} + \lim_{s\to -1}\frac{2}{s(s+4)}e^{st} + \lim_{s\to -4}\frac{2}{s(s+1)}e^{st}$$

$$\text{h(t)} = \frac{2}{4} - \frac{2}{3}e^{-t} + \frac{2}{12}e^{-4t} = \underline{\frac{1}{2} - \frac{2}{3}e^{-t} + \frac{1}{6}e^{-4t}}$$

Initial and final values of unit step response

$$\text{h(0)} = \lim_{t\to 0}\text{h(t)} = \lim_{s\to\infty}\text{s.H(s)} = \lim_{s\to\infty}\text{s.}\frac{G(s)}{s} = \lim_{s\to\infty}\text{G(s)} = \lim_{s\to\infty}\frac{2}{s^2+5s+4} = \frac{2}{\infty} = 0$$

$$\text{h}(\infty) = \lim_{t\to\infty}\text{h(t)} = \lim_{s\to 0}\text{s.H(s)} = \lim_{s\to 0}\text{s.}\frac{G(s)}{s} = \lim_{s\to 0}\text{G(s)} = \lim_{s\to 0}\frac{2}{s^2+5s+4} = \frac{2}{4} = 0{,}5$$

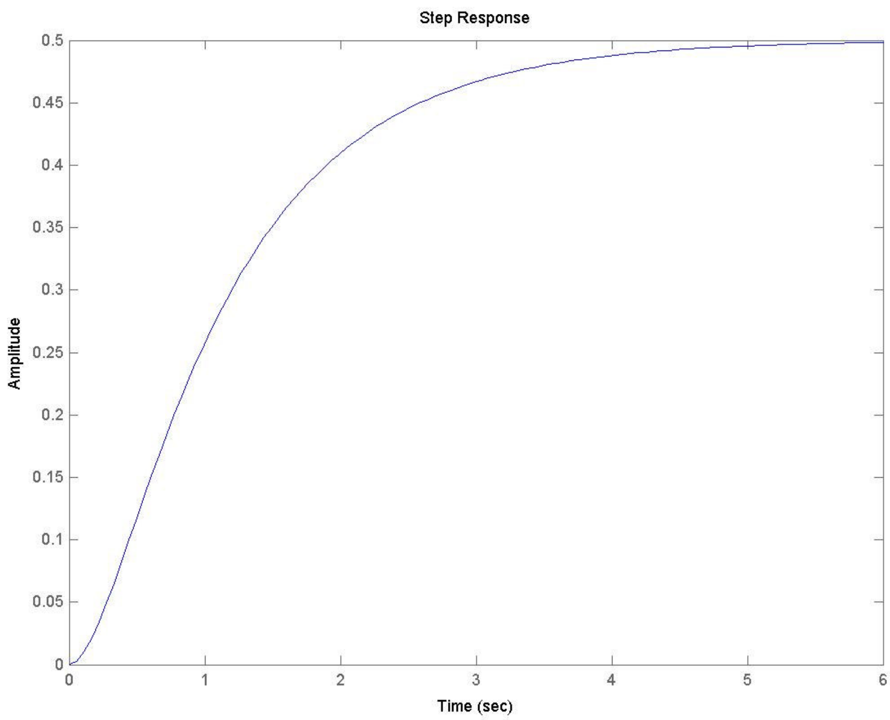


Figure 1. Unit step response, MATLAB

Command → step([2],[1 5 4])

5)

Impulse response

Impulse function is response to Dirac delta function at zero initial conditions

i(t) = h'(t) , i(t) = L^{-1}{G(s)} = $L^{-1}\left\{\dfrac{2}{s^2+5s+4}\right\}$

i(t) = $\lim\limits_{s\to -1}\dfrac{2}{s+4}e^{st} + \lim\limits_{s\to -4}\dfrac{2}{s+1}e^{st}$

i(t) = $\dfrac{2}{3}e^{-t} - \dfrac{2}{3}e^{-4t}$

Initial and final values of Impulse response

i(o) = $\lim\limits_{t\to 0}$ f(t) = $\lim\limits_{s\to\infty}$ s G(s) = $\lim\limits_{s\to\infty} s\cdot\dfrac{2}{s^2+5s+4} = \lim\limits_{s\to\infty}\dfrac{2}{2s+5} = 0$

i(o) = $\lim\limits_{t\to\infty}$ f(t) = $\lim\limits_{s\to 0}$ s G(s) = $\lim\limits_{s\to 0} s\cdot\dfrac{2}{s^2+5s+4} = 0$

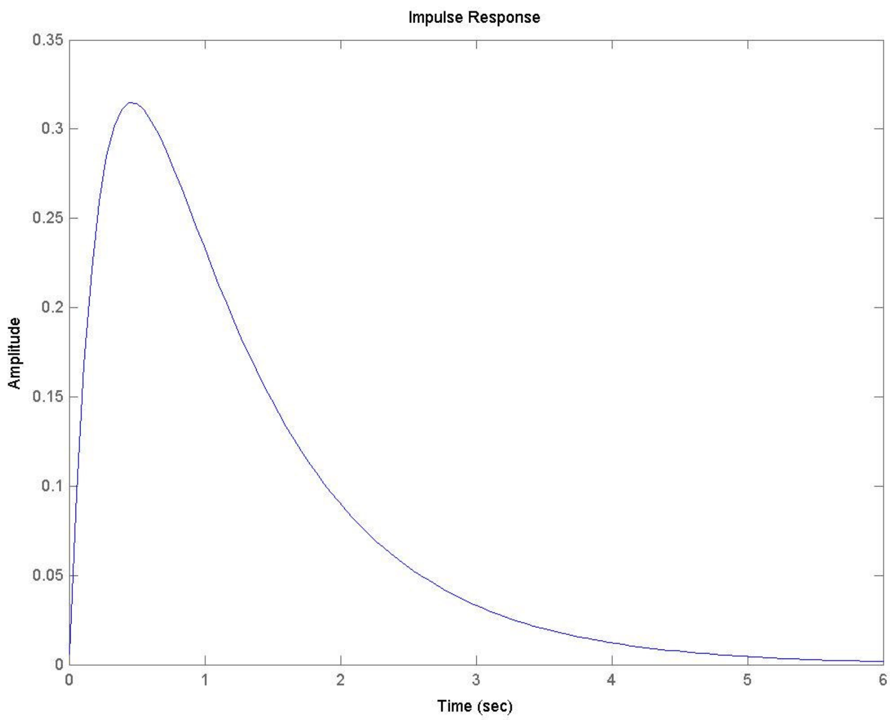


Figure 2. Impulse response, MATLAB

Command → impulse([2],[1 5 4])

6)

Amplitude-phase frequency response

Amplitude-phase frequency response is graphic display of frequency transfer in complex plane for ω ∈<0, ∞)

$$G(s) = \frac{2}{s^2 + 5s + 4}$$

$$G(j\omega) = \frac{2}{(j\varpi)^2 + 5j\varpi + 4} = \frac{2}{-\varpi^2 + 5j\varpi + 4} \cdot \frac{(4-\varpi^2) - 5j\varpi}{(4-\varpi^2) - 5j\varpi} = \frac{8 - 2\varpi^2 + 10j\varpi}{(4-\varpi^2)^2 + 5^2\varpi^2} =$$

$$= \frac{8 - 2\varpi^2}{\varpi^4 + 17\varpi^2 + 16} \cdot j\frac{-10\varpi}{\varpi^4 + 17\varpi^2 + 16}$$

Values on axis *x* are calculated from real part of transfer G(jω)

$$Re[G(j\omega)] = \frac{8 - 2\varpi^2}{\varpi^4 + 17\varpi^2 + 16}$$

Values on axis *y* are calculated from imaginary part of transfer G(jω)

$$Im[G(j\omega)] = \frac{-10\varpi}{\varpi^4 + 17\varpi^2 + 16} \cdot j$$

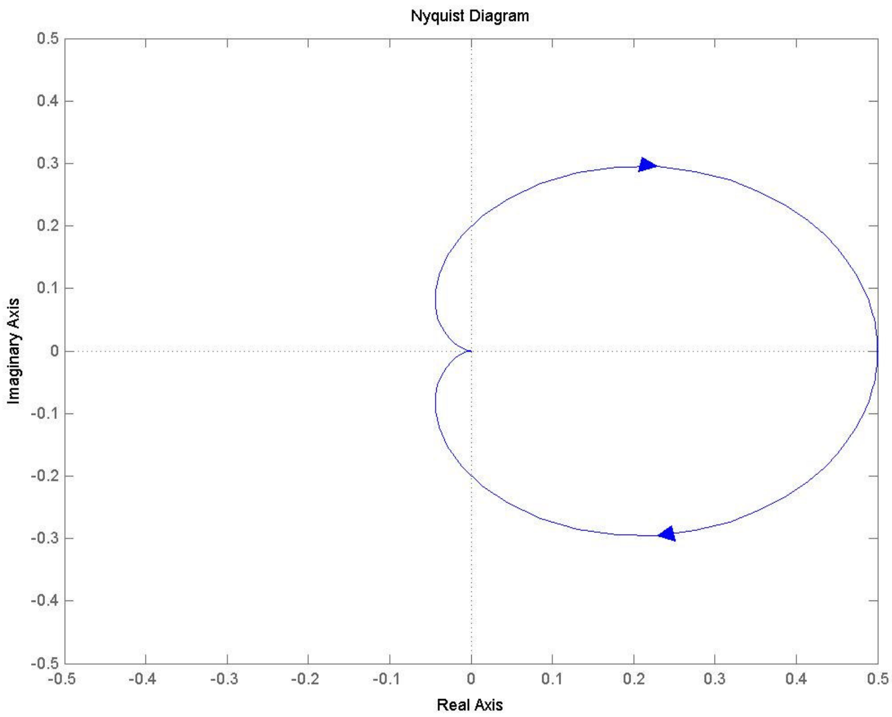


Figure 3. Nyquist diagram, MATLAB

Command → nyquist ([2],[1 5 4])

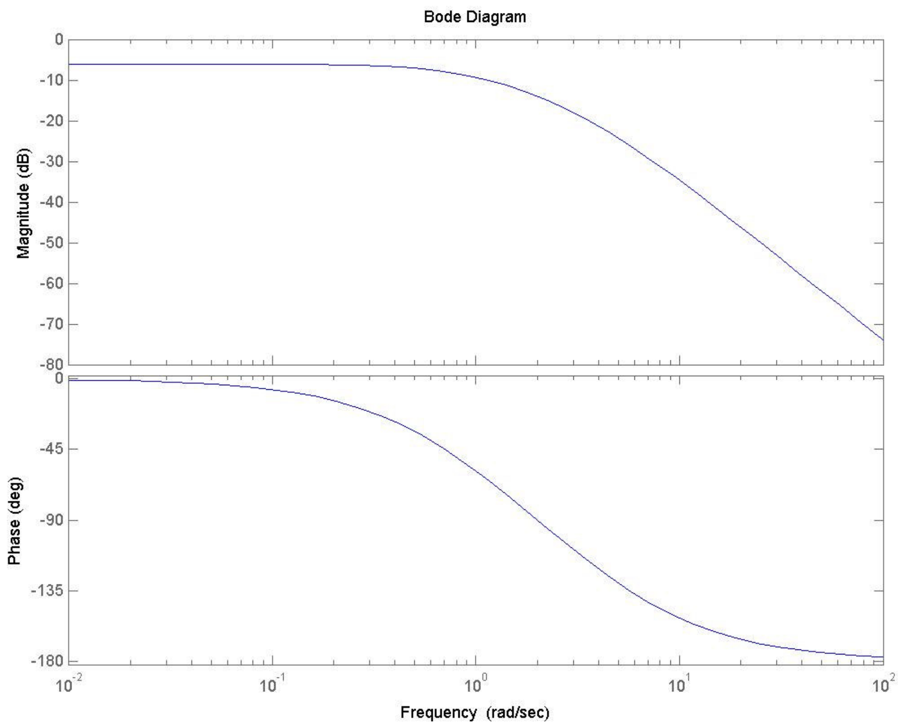


Figure 4. Bode diagram, MATLAB

Command → bode ([2],[1 5 4])

Conclusion:

From differential equation $y''(t)+5y'(t)+4y(t)=2u(t)$ was determined transfer

G(s) = $\frac{2}{s^2+5s+4}$

Poles of the transfer: -1, -4

Order and relative order: 2

Calculated unit step function:

h(t) = $\frac{1}{2}-\frac{2}{3}e^{-t}+\frac{1}{6}e^{-4t}$

Calculated impulse function:

i(t) = $\frac{2}{3}e^{-t}-\frac{2}{3}e^{-4t}$

The given system is stable (all of the poles lies in left part of complex plane), aperiodic (all of the poles lies on real axis) and minimum phase.

Graphs are depicted by the help of program MATLAB

9.2 Druhý protokol – syntéza regulačního obvodu

TOMAS BATA UNIVERSITY IN ZLIN Faculty of Applied informatics			
Name:		**Grade:**	II
Subject:	Automatic control theory	**Group:**	
Theme:	Continuous dynamic system – Synthesis of control system, description and analysis		

SISO (single input-output) linear continuous dynamic system is given by differential equation

$$a_2 y''(t) + a_1 y'(t) + a_0 y(t) = b_0 u(t)$$

Tasks:

1) Propose continuous controller by the help criterion stability and any classical method. Verify functionality and compare acquired results.

2) Propose controller by the help of polynomial synthesis for 1 DOF and 2 DOF configuration. Depicture control process for both configurations.

3) Suppose this system with time delay term in $\Theta \in \langle 1;10 \rangle$ and compare the simulation behaviour of the feedback loop with and without Smith's predictor. Then approximate the delay term and propose controller by the help of polynomial synthesis for 1DOF or 2DOF configuration.

Elaboration

1)

Settings value: $a_2 = 1$, $a_1 = 5$, $a_0 = 4$, $b_0 = 2$

Differential equation: $y''(t) + 5y'(t) + 4y(t) = 2u(t)$

Transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{2}{s^2 + 5s + 4} = \frac{2}{(s+1)(s+4)} = \frac{(s-n_1)}{(s-p_1)\cdot(s-p_2)}$$

$$G(s) = \frac{2}{s^2 + 5s + 4} = \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{b}{a} \qquad G(s) = \frac{Y}{U} = \frac{b}{a}$$

$$C(s) = r_0 + \frac{r_{-1}}{s} = \frac{r_0 \cdot s + r_{-1}}{s} = \frac{q_1 \cdot s + q_0}{s} = \frac{q}{p} \qquad C(s) = \frac{U}{E} = \frac{q}{p}$$

$ap + bq = s.(s^2 + 5s + 4) + (q_1 s + q_0)\cdot 2 = s^3 + 5s^2 + 4s + 2q_1 s + 2q_0 =$

$= \underline{s^3 + 5s^2 + (4 + 2q_1)\cdot s + 2q_0}$

Routh-Schure's criterion

1	5	$4+2q_1$	$2q_0$	
5		$2q_0$		/.(-0,2)
0	5	$4+2q_1 - 0,4$	$2q_0$	
	z_1	z_2	z_3	

$z_1>0$ z_1: $5>0$

$z_2>0$ z_2: $4+2q_1 - 0,4>0$

$z_3>0$ z_3: $2q_0>0$

$q0 = r\text{-}1 \Rightarrow 2$

$q_1 > \frac{0{,}4q_0 - 4}{2} = q_1 > \frac{0{,}8 - 4}{2} = -1{,}6 \Rightarrow q_1 = r_0 = 1{,}5$

$q_1 > -1{,}6$

$2 > -1{,}6 \qquad \Rightarrow \qquad C(s) = \frac{2s+2}{s}$

<u>Naslin method</u>

$$G(s) = \frac{2}{s^2 + 5s + 4} = \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{b}{a}$$

$$C(s) = r_0 + \frac{r_{-1}}{s} = \frac{r_0 \cdot s + r_{-1}}{s} = \frac{q_1 \cdot s + q_0}{s} = \frac{q}{p}$$

Overshoot: 1%

$\alpha = 2{,}4$

α	1,75	1,8	1,9	2	2,2	2,4
Δy_{max} [%]	16	12	8	5	3	1

Table 1 – Dependence Δy_{max} on α, by Naslin

<u>Characteristic equation</u>

$ap + bq = 0$

$s^3 + 5s^2 + (4 + 2q_1) \cdot s + 2q_0 = 0$

$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$

a^3: 1

a^2: 5

a^1: $4+2q_1$

a^0: $2q_0$

i = 1

$a_1^2 \geq \alpha \cdot a_0 \cdot a_2$

$(4+2q_1) \geq 2{,}4 \,.\, 2q_0 \,.\, 5$

$(4+4)^2 \geq 24q_0$

$2{,}67 \geq q_0 \quad \Rightarrow q_0 = 2$

i = 2

$a_2^2 \geq \alpha \cdot a_1 \cdot a_3$

$5^2 \geq 2{,}4 \,.\, (4+2q_1) \,.\, 1$

$25 \geq 9{,}6 + 4{,}8\, q_1$

$$\frac{15{,}4}{4{,}8} \geq q_1$$

$3{,}2083 \geq q_1 \quad \Rightarrow q_1 = 2$

$$C(s) = \frac{2s+2}{s}$$

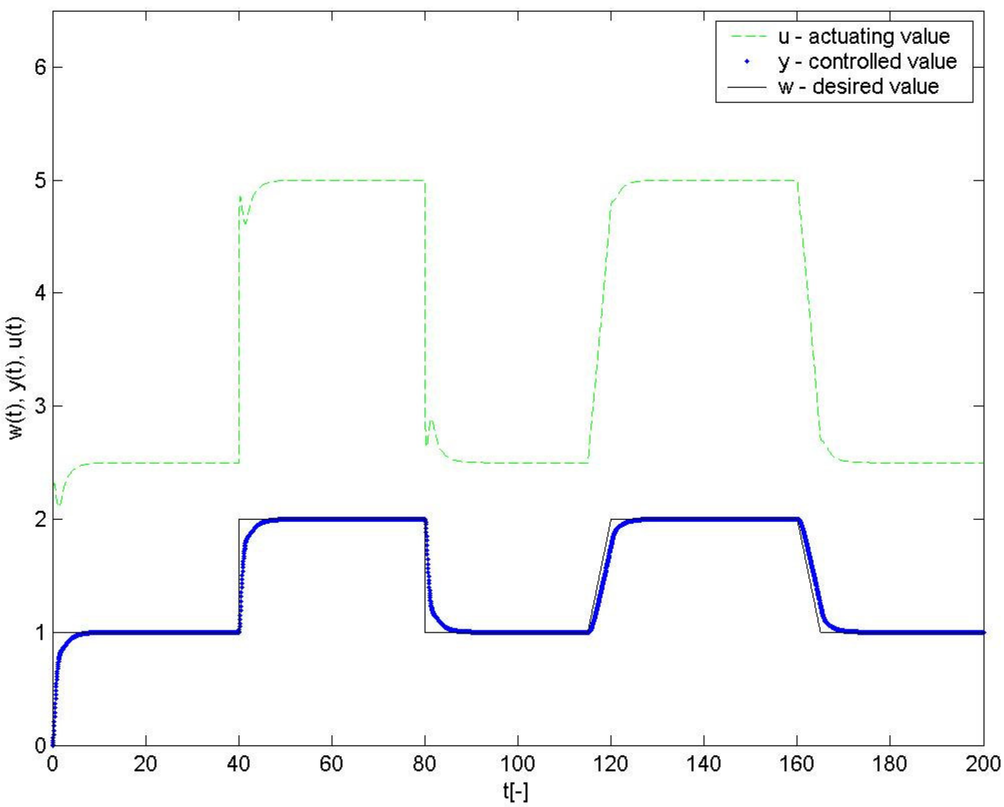
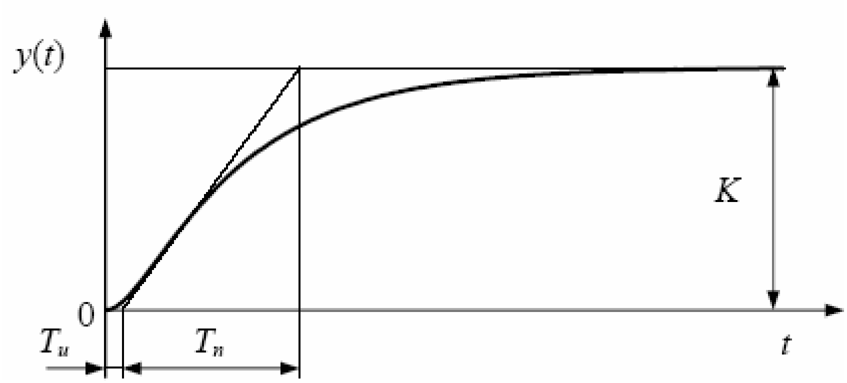


Figure 1 - Behaviour regulation of given transfer function G(s), parameters of controller C(s) were adjusted by the help Naslin method.

Setting value from unit step response



$$C(s) = k_P\left(1 + \frac{1}{T_I s} + T_D s\right), \text{ or } C(s) = r_0 + \frac{r_{-1}}{s} + r_1 s$$

Parameters Tu, Tn, K was deducted from unit step response by the help software Matlab

Tu = 0,1246

Tn = 1,5878

K = 0,4999 $\gamma = \frac{Tn}{Tu} = \frac{1{,}5878}{0{,}1246} = 12{,}74$

	k_P	T_I	T_D
P	$\gamma\frac{1}{K}$	-	-
PI	$0{,}9\gamma\frac{1}{K}$	$3{,}5\ T_u$	-
PD	$1{,}2\gamma\frac{1}{K}$	-	$0{,}25T_u$
PID	$1{,}25\gamma\frac{1}{K}$	$2\ T_u$	$0{,}5\ T_u$

Table. 2 Table of transfer relations for calculation parameters

PI :

$k_p = 0{,}9\gamma\frac{1}{K} = 0{,}9 \cdot 12{,}74 \cdot \frac{1}{0{,}4999} = 22{,}94$

$T_I = 3{,}5 \cdot Tu = 3{,}5 \cdot 0{,}1246 = 0{,}4361$

Transfer function $C(s) = 22{,}94\left(1 + \frac{1}{0{,}44s}\right)$

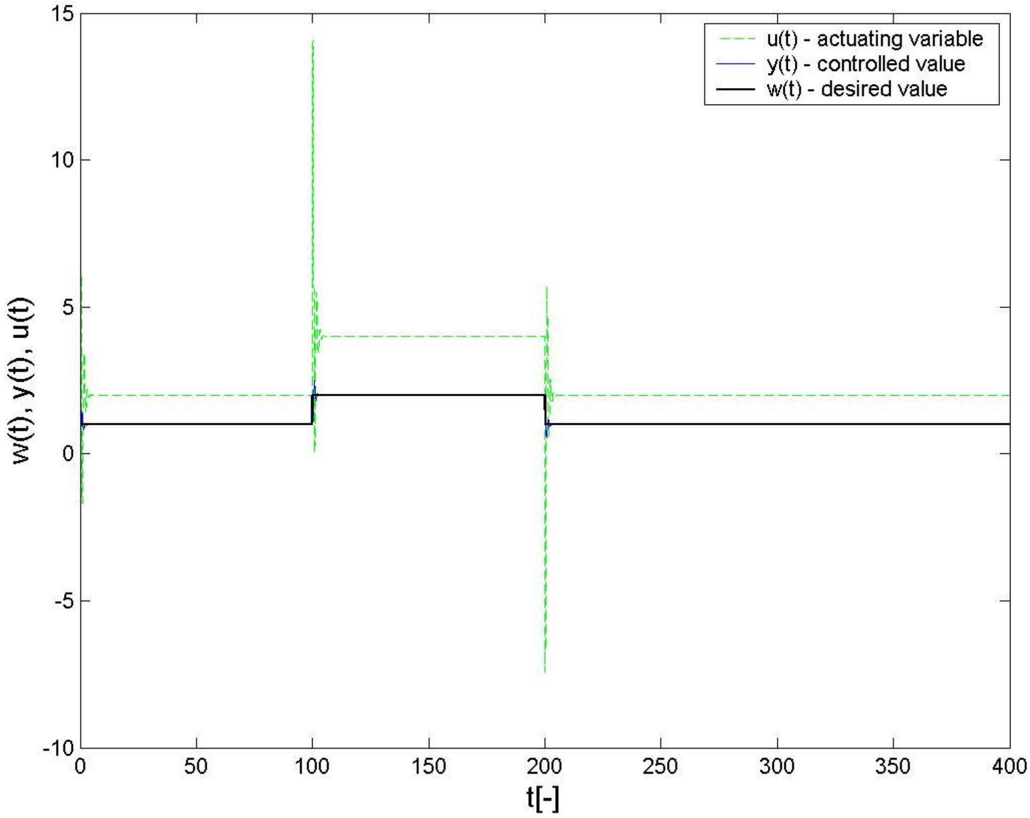


Figure 2 - Behaviour regulation of given transfer function G(s), parameters of controller C(s) were adjusted from unit step response

The rest of method and conclusion are included in CD insertion.

9.3 Třetí protokol – stavový popis

TOMAS BATA UNIVERSITY IN ZLIN Faculty of Applied informatics			
Name:		**Grade:**	II
Subject:	Automatic control theory	**Group:**	
Theme:	State space description LCDS		

SISO (single input-output) linear continuous dynamic system is given by differential equation

$$a_2 y''(t) + a_1 y'(t) + a_0 y(t) = b_0 u(t)$$

Tasks:

1) Determine state space description of engaged control system. Make transmission from inner description to outer description.

2) Determine controllability matrix and observability matrix and decide if the system is controllable or observable.

Elaboration

1)

Settings value: $a_2 = 1$, $a_1 = 5$, $a_0 = 4$, $b_0 = 2$

Differential equation: $y''(t) + 5y'(t) + 4y(t) = 2u(t)$

Transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{2}{s^2 + 5s + 4} = \frac{2}{(s+1)(s+4)} = \frac{(s-n_1)}{(s-p_1)\cdot(s-p_2)}$$

$$G_S(s) = \frac{Y(s)}{U(s)} = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} = \frac{1 \cdot s + 2}{s^2 + 5s + 4}$$

$$G_S(s) = \frac{Y(s)}{Z(s)} \cdot \frac{Z(s)}{U(s)} = \frac{1 \cdot s + 2}{1} \cdot \frac{1}{s^2 + 5s + 4}$$

First part of transfer function

$$\frac{Y(s)}{Z(s)} = 1.s + 2$$

Differential equation

$y(t) = 1.z'(t) + 2.z(t)$

Second part of transfer function

$$\frac{Z(s)}{U(s)} = \frac{1}{s^2 + 5s + 4}$$

Differential equation

$u(t) = 1.z''(t) + 5.z'(t) + 4.z(t)$

Choosing state space variables

$x_1 = z$

$x_2 = z'$

Differential equations of 1st order

$x_1' = x_2$

$x_2' = z'' = u(t) - 5 \cdot z'(t) - 4 \cdot z(t) = u - 5 \cdot x_2 - 4 \cdot x_1$

Output equation

$y(t) = 1 \cdot x_2 + 2 \cdot x_1$

State space model

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot u$$

$$y = \begin{pmatrix} 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + (0) \cdot u$$

Conversion state space description to transfer function

$$A=\begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} \qquad B=\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad C=\begin{pmatrix} 2 & 1 \end{pmatrix} \qquad D=(0)$$

$$G(s) = C\cdot(s\cdot I - A)^{-1}\cdot B + D$$

$$G(s) = \begin{pmatrix} 2 & 1 \end{pmatrix} \cdot \left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix}\right]^{-1} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} + (0) =$$

$$= \begin{pmatrix} 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} s & -1 \\ 4 & s+5 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \frac{1}{s^2+5s+4} = \frac{1\cdot s+2}{s^2+5s+4}$$

2)

Controllability matrix

$$R = \begin{pmatrix} B & A\cdot B \end{pmatrix}$$

$$A\cdot B = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 1 \\ 1 & -5 \end{pmatrix} \qquad \Rightarrow \text{rank}(R) = 2$$

rank(A) = 2

det = -1 $\neq$ 0

Rank of controllability matrix is equal degree of transfer; determinant of controllability matrix isn't equal zero $\Rightarrow$ system is controllable

Observability matrix

$$P = \begin{pmatrix} C \\ A\cdot C \end{pmatrix}$$

$$A\cdot C = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \end{pmatrix} = \begin{pmatrix} -4 & -3 \end{pmatrix}$$

$$P = \begin{pmatrix} 2 & 1 \\ -4 & -3 \end{pmatrix} \qquad \Rightarrow \text{rank}(P) = 2$$

rank(A) = 2

det = -2 $\neq$ 0

Rank of observability matrix is equal with degree of transfer, determinant of observability matrix isn't equal zero → system is observable

Conclusion

From the given system was determined state space description, and then was verify parameters of state space description. Also was made transmission from inner description to outer description. From transfer function was determined parameters:

$A=\begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix}$ $B=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $C=\begin{pmatrix} 2 & 1 \end{pmatrix}$ D= (0)

Controllability matrix

$P = \begin{pmatrix} 2 & 1 \\ -4 & -3 \end{pmatrix}$

Observability matrix

$R = \begin{pmatrix} 0 & 1 \\ 1 & -5 \end{pmatrix}$

In terms of these matrixes was decided, that the given system is controllable and observable.

ZÁVĚR

Cílem této bakalářské práce bylo vytvořit studijní návody a opory pro předmět Teorie automatického řízení I. Materiály jsou v anglickém jazyce a budou sloužit pro zahraniční studenty. Teoretická část je zaměřena na stručný popis látky, která je podrobněji popsána v prezentacích vytvořených v programu PowerPoint. V praktické části jsou zobrazeny samotné prezentace v rozložení 6 snímků na list, dále jsou zde vzorové protokoly, které jsou rozděleny do tří částí. První část je zaměřena na vnější popis a analýzu lineárních spojitých dynamických systémů. Druhá část se zabývá syntézou regulačního obvodu a ve třetí je úkolem studentů určit stavový popis lineárních spojitých dynamických systémů. Všechny materiály budou k dispozici ke stažení z univerzitního webu. Simulace byly provedeny pomocí programu MATLAB/SIMULINK.

ZÁVĚR V ANGLIČTINĚ

Aim of this bachelor work was to create study instructions and supports for subject Theory of automatic control. This study is in english language and is instrumental for foreign students. Theoretical part is concentrated to brief description of substance that is more described in presentations of PowerPoint. In practical part are displayed the presentations in group of six pictures. Next there are exemplary protocols that are divided up into three parts. First part is focused on outer description and analysis linear continuous dynamic systems. Second part contains synthesis of control system and third part is oriented to state space description of linear dynamic continuous systems. All materials will be possible download from university web. Simulations were made by the help of program MATLAB/SIMULINK.

SEZNAM POUŽITÉ LITERATURY

[1] BALÁTĚ, J. *Automatické řízení*. Praha : BEN-technická literatura, 2003. 664 s. ISBN 80-7300-020-2.

[2] VÍTEČKOVÁ, M., VÍTEČEK, A. *Základy automatické regulace*. [s.l.] : VŠB-OSTRAVA, 2006. 200 s. ISBN 80-248-1068-9.

[3] VÍTEČKOVÁ, M., VÍTEČEK, A. *Anglicko-český slovník základních pojmů z oblasti automatického řízení*. [s.l.] : VŠB-TECHNICKÁ UNIVERZITA OSTRAVA, 2006. 96 s. ISBN 80-248-1069-7.

[4] *Automatizace* [online]. 2007 [cit. 2008-05-15]. Dostupný z WWW: <http://www.e-automatizace.cz>.

[5] *Automatizace* [online]. 2007 [cit. 2008-05-18]. Dostupný z WWW: <http://www.caac.zde.cz>.

[6] MIDDLETON, R.H., GOODWIN, G.C. *Digital control and estimation a unified approach*. [s.l.] : Prentice-Hall, INC., 1990. 538 s. ISBN 0-13-211798-3.

SEZNAM POUŽITÝCH SYMBOLŮ A ZKRATEK

LSDS - lineární spojitý dynamický systém

$G(s)$ - přenos soustavy

$Q(s)$ - přenos regulátoru

$F(s)$ - obraz Laplaceovy transformace

$f(t)$ - originál Laplaceovy transformace

$u(t)$ - vstupní veličina

$y(t)$ - výstupní veličina

$h(t)$ - přechodová funkce

$i(t)$ - impulsní funkce

$G(j\varpi)$ - frekvenční přenos

s_i - kořeny jmenovatele (póly)

p_i - kořeny čitatele (nuly)

P - proporcionální složka regulátoru

I - integrační složka regulátoru

D - derivační složka regulátoru

T_I - integrační časová konstanta

T_D - derivační časová konstanta

T_u - doba průtahu

T_n - doba náběhu

K - zesílení

SEZNAM OBRÁZKŮ

SEZNAM PŘÍLOH

Příloha 1: 1 ks CD-ROM

PŘÍLOHA P I: CD-ROM

Obsahuje tyto adresáře:

BAKALÁŘSKÁ PRÁCE

PREZENTACE

VZOROVÉ PROTOKOLY