Tomas Bata University in Zlín
Faculty of Applied Informatics

Nonlinear System Identification and Control Using Local Model Networks

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Nomenclature

Acronyms

ARX	AutoRegressive with eXogeneous inputs
ASMOD	Adaptive Spline Modelling of Observation Data
BFGS	Broyden-Fletcher-Goldfarb-Shanno method
CSTR	Continuous Stirred Tank Reactor
GMV	Generalized Minimum Variance
EM	Expectation Maximization algorithm
J&F	construction algorithm of Johansen and Foss
LLC	Local Linear Controller
LLM	Local Linear Model
LMN	Local Model Network
LOLIMOT	Local Linear Model Tree
MLP	Multiple Layer Perceptron
MPC	Model Predictive Control
MMPC	Multiple Model Predictive Control
NARX	Nonlinear Auto Regressive with eXogenous input
QP	Quadratic Programming
RBF	Radial Basis Function
RLS	Recursive Least-Squares
SVD	Singular Value Decomposition

General Notations

a, b, θ	Scalars
$\boldsymbol{a, b, \theta}$	Vectors
$\boldsymbol{A, B}, \ldots$	Matrices
$A(z^{-}), B(s)$	Polynomials

Functions and Operators

Variables

$\boldsymbol{a}$	parameter vector of the local model
A	denominator polynomial of a system
$\boldsymbol{b}$	parameter vector of the local model
B	numerator polynomial of a system
c	centre of the validity function
e	estimation error
H_c	control horizon
H_p	prediction horizon
J	objective or cost function
M	number of models
n_a	number of maximum lags in output signal
n_b	number of maximum lags in input signal
σ	width of the validity function
$\hat{\boldsymbol{\theta}}$	vector of parameter estimates
ρ	normalised validity function

$\tilde{\rho}$	unnormalised validity function
$\boldsymbol{\varphi}$	regression vector
$\boldsymbol{\psi}$	vector of scheduling variables

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Chapter 1 INTRODUCTION

1.1 Background

Technology development is constantly bringing more complex production facilities and thus raises the need of appropriate tool for engineers to help them understand and solve such systems. In everyday life, the strategy how to solve a complex problem is called divide & conquer. The problem is divided into simpler parts, which are solved independently and together yields the solution to the whole problem. The same strategy can be used for control of non-linear systems, where the non-linear plant is substituted by locally valid set of linear sub models. The model should satisfy two criteria: it must be simple enough so that it can be easy understood and complex enough in order to provide accurate predictions. The accurate model that characterizes important aspects of the system being controlled is a necessary prerequisite for design of a controller. Therefore, system identification has become a key issue in the control literature.

To accurately model the nonlinear system, a wide variety of techniques has been developed such as nonlinear autoregressive moving average with exogenous inputs (NARMAX) models [1], Hammerstein models [2] or Multiple Layer Perceptron (MLP) neural network [3]. Even though, these methods offer improved accuracy over a single linear model, the black box representation of dynamics in these methods fails to exploit the theoretical results available in the conventional linear modelling and control domain. Moreover, the black-box representation of MLP networks lacks transparency. Besides MLP networks, Radial Basis Function (RBF) networks, which were initially introduced for multivariable interpolation, are other popular neural networks. They have been successfully applied in the fields of aerospace, robotics, power generation and chemical manufacturing.

The Multiple Model approach that utilizes different models for different operating points offers both transparency and possibility to include the *a priori* knowledge about the system [4]. The Multiple models method appears in the literature under many

different names, including local model networks or operating regime decomposition. The idea of approximation based on multiple models (MM) is not new. The piece-wise linear approximations [6], which use the set of local models and switching, clearly fall into this category. Fuzzy identification approach [7] is probably the first example of the piecewise model with soft transitions. Due to the structure of the local model networks and the similarity to the neural networks, it is hard to decide whether local model networks belong to the category of fuzzy systems or neural networks.

Local Model Networks are networks which are composed of locally accurate models, whose output is interpolated by smooth, locally active validity functions. This divide-and-conquer strategy is a general way of coping with complex systems. The architecture of LMN benefits from being able to incorporate the *a priori* knowledge about the system and conventional system identification methodology. The LMN structure also gives transparent and simple representation of the nonlinear system. Contrary to the black box representation of the nonlinear process by the neural networks, the conventional design methods can be utilized for nonlinear controller design. The idea of the LMN approach is to split the whole operating region into several sub-regions where in each sub-region the process has close to linear behaviour. For each region, a local linear model is developed to approximate the non-linear dynamics and associated with a validity function. This function can be viewed as a weighting function of the local models. The value of the validity function is high if the input vector lies inside the operating region and decreases with distance between the input vector and the centre of the region. The main feature of the validity function is to blend local models to give a nonlinear approximation of the system. During the learning algorithm, the local models, validity functions and the form of their blending have to be determined.

There is no specific method to determine correctly the structure and parameters of the LMN, however, the following issues should be concerned:

- The structure and number of the local models
- The division of the operating range into regimes
- Interpolation among the local models

Variety of different methods and algorithms for structure optimization has been developed. The task of training can be divided into two parts: structure optimization

and local model parameters estimation. Structure optimization comprises the determination of the shapes of the operating regions, i.e. the number of the local models, their type and parameters of the validity functions. The Expectation Maximization [8] algorithm is usually used for the Gaussian process models although it requires the prior knowledge of complexity of the system or more precisely the number of local models. Xue and Li in [9] developed the Satisfying Fuzzy c-mean Clustering Algorithm which adds a new cluster centre if the modelling performance index, defined by Root Mean Squared Error, is not satisfied. Methods developed by Johansen and Foss, [5], Nelles, [10] start with a single model and hierarchically partition operating space and iteratively increase the number of models and thus preventing from overfitting. McLoone *et al*. in [11] proposed an off-line hybrid training method for LMN which combines the full memory Broyden-Fletcher-Goldfarb-Shanno method (BFGS) for estimating nonlinear parameters and the linear LSM for linear weight estimation of local model parameters. Sharma, McLoone and Irwin in [12] describe identification of both the validity functions and local model parameters using the genetic learning approach. Other methods are also mentioned in the Structure Identification section of the thesis.

As employed local models are usually linear, after the parameters of validity functions are determined the problem is reduced to a simple linear optimization problem, and thus can be solved by the standard least-squares or weighted least-squared method. Two different learning methods can be implemented for parameter computation. Global learning is based on the assumption that all the parameters are estimated in a single regression, which is not always computationally feasible for problems with a large number of data. An alternative to global learning, which estimates the parameters of each local model independently is local modelling. Due to local modelling, local models can be interpreted independently and can be seen as local approximations of the nonlinear system. The comparison of both methods can be found in [13]. To achieve good trade-off in terms of global fitting and local linearization weighted performance index can be defined as in [9].

Once the LM network has been formulated, the local controller network can be defined in turn. Since the neural network can accurately model the nonlinear dynamics, the local model network can be implemented in the control structures that demand the

predictions of future outputs, such as predictive control [14] or internal model control [15], [16]. Two different multiple model controller design methods can be employed to maintain the performance. In one case, a controller is designed for each local model and the control action is then weighted according to the value of validity functions [17]. Dougherty and Cooper developed a multiple model control strategy for dynamic matrix control (DMC), where outputs of multiple linear DMC controllers are weighted to obtain an adaptive DMC controller [18]. Li *et al*. in [19] proposed a Multiple Model Predictive Control (MMPC) which uses different predictive controllers for different fuzzy rules of the Takagi-Sugeno model of the process. The use of local model networks is not limited only to predictive control. Brown and Irwin in [20] used local GMV controllers to form a nonlinear controller network. The major advantage of this approach is that each controller can be designed using different technique known from linear theory, i.e. pole placement, GMV, LQ etc. For the other case, a single controller is used. The process models were scheduled using the process variable measurement and the resulted model is used to design a controller [21]. Abonyi *et al*, in [22] proposed a model-based predictive controller that is based on a local linear approximation of the fuzzy-based process model around the current operating point. Dharaskar and Gupta in [23] use the interpolated step responses that are easy to obtain to handle the nonlinearity of the chemical processes. Townsend *et al*. in [21] developed a control structure, where the process model was substituted by the local model network. This LMN contained a set of ARX models and had been trained by hybrid training algorithm. Narendra and Xiang in [24] designed multiple model controllers using both fixed and adaptive models. Criterion based on the prediction errors of models is used for switching between the controllers.

The Multiple Model control does not have to imply blending or interpolation. The use of local linear models without interpolation has been suggested by several authors. The simplest form of scheduling is hard switching between controllers or control parameters. The switching algorithm (controller selection) is usually based on the modelling error where model with the lowest modelling error initializes corresponding controller. The idea has been used in [25] to develop the Multiple Switched Model (MSM) control scheme. This control scheme is capable of handling plants with rapid change of parameters. The modelling error is used as a criterion for controller selection

which can be justified by the fact that a controller is highly dependent on a plant model. The stability and robustness description can be found in [24]. The switching between the controllers is based on “hard” transitions between the models or controllers. The local controllers can be also associated with processing element of a self-organizing map (SOM) which is trained using the input-output data [26]. Banerjee et al. merged multiple models into a global one by probability integration, and used probability as a scheduling variable to select proper controller [27].

1.2 Aims of thesis

The LMNs provide models with high accuracy and transparency. The model can be easily interpreted in the terms of physical system. Moreover, *a priori* information can be included during the structure development and parameter estimation. One of the aims of this work is development and investigation of various techniques for optimization of the local model network parameters, i.e. parameters of local models and validity function. Experimental identification from input/output data should utilize affine local ARX models. Since the main application for nonlinear dynamics model is dynamic optimization and predictive control, the other aim is to investigate various control algorithms that employ the local model network as a nonlinear model for control output computation. In order to examine the practical applicability of the local model network the modelling and control algorithms are tested in several simulation studies and laboratory experiments.

1.3 Overview of thesis

The thesis is organized as follows. In **Chapter 1** the background concerning the multiple model control and identification is given. **Chapter 2** gives introductory details of local model networks structure and shows how it can be viewed as a general case of the Radial Basis Function (RBF) neural network. Different forms of operating regions and validity functions are presented.

In **Chapter 3** the training approaches for local model networks are summarized. This chapter also deals with the structure identification of nonlinear systems from input/output data. Main differences between local and global training techniques are pointed out. The algorithm of Johansen and Foss as a typical example of heuristic strategy with growing number of models is described. Structure identification using evolutionary algorithm that offers easy incorporation of constraints in relation to transparency and interpretability is mentioned. Parameter optimization method that implements Quadratic Programming (QP) to include the constraints on model parameters is presented. **Chapter 4** deals with the model based control algorithms based on local model description of the nonlinear system. The first scheme used in this thesis is a predictive control scheme, which uses a local model network to predict the future trajectory of the controlled variable. The LMN also provides the parameters of the plant on the future trajectory. The second scheme is the Internal Model Control scheme that uses the LMN as the model of a process and linearization of LMN for analytical inversion of the model.

In **Chapter 5** experimental results including simulation studies on pH neutralization plant and experiments on laboratory plants are presented.

Chapter 6 finishes the thesis with some general conclusions and suggestions for future work in this rapidly expanding field.

Chapter 2 LOCAL MODEL NETWORKS

This chapter begins with the description of the Radial Basis Function neural networks and shows the connection with the Local Model Network structure. The chapter continues with an overview of the basic structure of local model network where specifications of validity functions and local models are given. The similarity between LMNs and Takagi-Sugeno fuzzy models is explained. The problems connected with normalization of the validity functions and purely local linear models are also mentioned.

2.1 Introduction

There exists mismatch between the available theoretical tool and most of the problems encountered in practice. While plenty of theoretical analysis and methods have been developed for linear systems, the practitioners are confronted with systems with apparent nonlinearities of the real world. An appealing approach to bridge the existing gap is to decompose a complex nonlinear control problem in a number of simpler linear problems, each associated with restricted operating region. Local Model Network employs this divide-and–conquer strategy of dividing a complex problem into several simple sub-problems, whose individual solution yields the solution to the complex problem. Local model network [4] belongs to the class of multiple model approaches with interpolation, wherein a small number of relatively simple models is blended together. Typically, each local linear or affine model is associated with a corresponding weighting function that defines the validity of the model. The role of blending is to provide smooth interpolation between the outputs of local models. The LMN framework provides transparency and enables incorporation of *a priori*

knowledge, which is important for practical applications. The resulting nonlinear representation is either discrete or continuous-time depending on the form of the local models. Although much more attention to date has been given to the discrete-time models, several works concerning the continuous-time case can be found in [29][30].

2.2 Radial Basis Network

RBF networks were introduced into the neural network literature by Broomhead and Lowe in [31] and Poggio and Girosi in [32]. It was soon discovered that this new architecture had a number of important advantages over the Multilayer Perceptron (MLP), in terms of training and locality of approximation. Hence, interest in the network grew rapidly and it became widely used as an alternative to MLPs for many modelling and control problems.

The RBF network is a three-layer feed-forward network that uses a linear transfer function for the output units and a nonlinear transfer function (normally the Gaussian) for the hidden units.

Many different types of nonlinearity have been proposed as basis functions. Those with a strong theoretical backing include the Gaussian, thin-plate spline, multiquadratic and inverse multiquadratic functions. Radial functions are special types of functions. Their characteristic feature is that their response decreases or increases monotonically with distance from the central point. The centre, the distance scale, and the precise shape of the radial functions are parameters of the model. A typical radial function is the Gaussian function, which in case of a scalar input is given,

$$\rho(x) = \exp\left(\frac{-\|x - c_i\|^2}{2\sigma_i^2} \right) \tag{2.1}$$

where the parameters of the Gaussian function are its centre c_i and width σ_i. The Gaussian function responds only to a small region of the input space where the Gaussian function is centred. The key to successful implementation of these networks is to find suitable centre for the Gaussian functions.

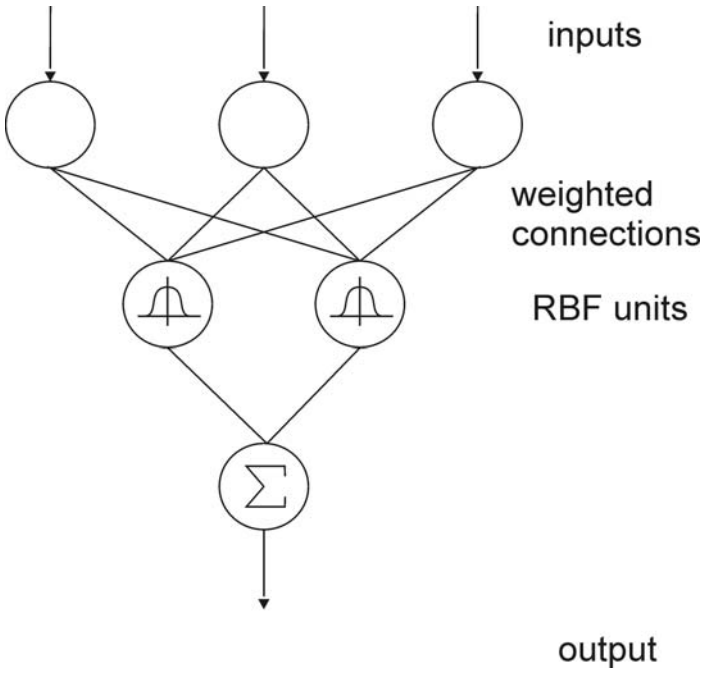


Figure 1 RBF network

Figure 1 illustrates an RBF network with 3 inputs, 2 neurons and 1 output. The RBF network consists of one hidden layer of basis functions, or neurons. At the input of each neuron, the distance between the neuron centre and the input vector is calculated. The output of the neuron is then formed by applying the basis function to this distance. The RBF network output is formed by a weighted sum of the neuron outputs and the unity bias shown.

RBF networks can be used to solve a common set of problems such as

- Function approximation
- Classification
- Modelling of dynamic systems and time series

2.3 RBF-ARX Model

In model based control strategies for nonlinear systems, RBF networks provide a framework for modelling the system to be controlled. However, in many real applications many RBF centres are needed for required precision which leads to the problems in parameter estimation. State-dependent AutoRegresive model with eXogenous variable (ARX) can be often used for modelling complex nonlinear systems. Connection of ARX models with the RBF network has the advantages of both the state-dependent ARX models for description of dynamics of the system and the RBF networks in function approximation. In general RBF-ARX network uses far fewer RBF centres when compared with a standard RBF network model.

The nonlinear system can be described by the following nonlinear ARX model

$$\begin{aligned} y(k) &= f(\boldsymbol{\varphi}(k-1)) + e(k) \\ \boldsymbol{\varphi}(k-1) &= [y(k-1),....y(k-n_a),u(k-1),...u(k-n_b)] \end{aligned} \tag{2.2}$$

where $y(k)$ is the output, $u(k)$ is the input, *e(k)* is the white noise and $\boldsymbol{\varphi}(k-1)$ is a regression vector. Various kinds of function can be used to approximate the nonlinear function $f(.)$. A general version is the state-dependent ARX model.

$$y(k) = \varphi_0(\boldsymbol{\varphi}(k-1)) + \sum_{i=1}^{n_a} \varphi_{y,i}(\boldsymbol{\varphi}(k-1)) y(k-i) + \sum_{i=1}^{n_b} \varphi_{u,i}(\boldsymbol{\varphi}(k-1)) u(k-i) + e(k) \tag{2.3}$$

where $\varphi_0, \varphi_{y,i}, \varphi_{u,i}$ are the coefficients which depend on the vector $\boldsymbol{\varphi}(k-1)$. Here, $\boldsymbol{\varphi}(k-1)$ is regarded as vector of past input and output values at the step *k*, but in many cases $\boldsymbol{\varphi}(k-1)$ can be only output signal, input signal or any other measured signal. The basic idea of the state-dependent ARX model is to provide the local linearization of the general NARX model. The RBF network can be used to approximate the state-dependent coefficient because it can approximate any function. The model derived is called the RBF-ARX model and can be defined by

$$
\begin{aligned}
y(k) &= \varphi_0(\boldsymbol{\varphi}(k-1)) + \sum_{i=1}^{n_y} \varphi_{y,i}(\boldsymbol{\varphi}(k-1)) y(k-i) + \sum_{i=1}^{n_u} \varphi_{u,i}(\boldsymbol{\varphi}(k-1)) u(k-i) + e(k) \\
\varphi_0(\boldsymbol{\varphi}(k-1)) &= c_0^0 + \sum_{k=1}^{m} c_k^0 \exp(-\lambda_k^y \left\| \boldsymbol{\varphi}(k-1) - Z_k^y \right\|_2^2) \\
\varphi_{y,i}(\boldsymbol{\varphi}(k-1)) &= c_{i,0}^y + \sum_{k=1}^{m} c_{i,k}^y \exp(-\lambda_k^y \left\| \boldsymbol{\varphi}(k-1) - Z_k^y \right\|_2^2) \\
\varphi_{u,i}(\boldsymbol{\varphi}(k-1)) &= c_{i,0}^u + \sum_{k=1}^{m} c_{i,k}^u \exp(-\lambda_k^u \left\| \boldsymbol{\varphi}(k-1) - Z_k^u \right\|_2^2)
\end{aligned}
\tag{2.4}
$$

where $Z_k^i (k = 1..m, i = y,u)$ are the centres of RBF networks; $\lambda_k^i (k = 1..m, i = y,u)$ are the scaling factors; $\varphi_0, \varphi_{u,i}, \varphi_{y,i}$ are the state-dependent coefficients; $c_k^0 (k = 0,1,..m), c_{i,k}^y (i = 1,2,...n_y; k = 0,1,..m), c_{i,k}^u (i = 1,2,...n_u; k = 0,1,..m)$ are the scalar constants and $\| \|_2$ denotes the vector 2-norm. In general case, the RBF network may have a different centre for different regressive variables. However, in some applications the RBF networks can share the same centres because of the autoregressive structure of the ARX model. The locally linearized model can be easily obtained by fixing the state vector $\boldsymbol{\varphi}(k-1)$. This property is very useful because it allows using linear model-based control method to design a controller, which cannot be done when using RBF network. In some applications of RBF networks, a large number of centres is necessary to obtain a good representation of the nonlinear system, which results in overfitting. However, the RBF-ARX model transfers the complexity into ARX part, so far fewer centres is necessary to obtain similar modelling properties.

2.4 Local Model Networks

Local Model networks (LMN), first introduced by Johansen and Foss [5], describe a set of sub-models, each valid for a specific regime in the operating space, weighted by some activation function. LMN is generalization of the radial basis function network (RBF), in which individual neurons are replaced by local submodels with basis functions defining the regions of validity of individual submodels, according to the expected operating regions of the plant [4].

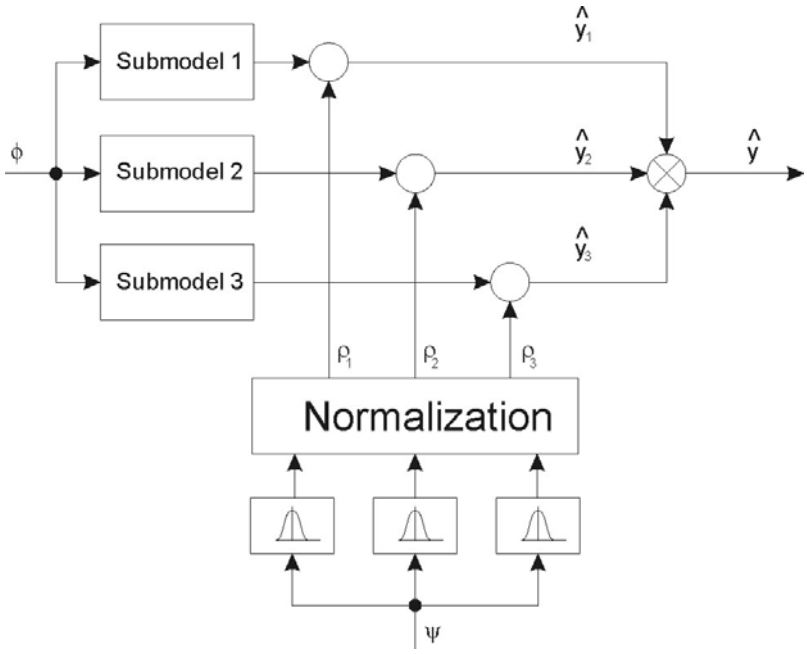


Figure 2 Local Model Network scheme

Here, the same input, $\varphi(k)$, is fed to all the models and the outputs are weighted according to some scheduling variable or variables, $\boldsymbol{\psi}$. The underlying local models can be either linear or nonlinear. The LM network output is given by:

$$\hat{y}(k) = \sum_{i=1}^{M} \rho_i(\boldsymbol{\psi}(k))\hat{y}_i(k) \tag{2.5}$$

where $\boldsymbol{\psi}$ is a vector of scheduling variables, $\rho_i(\boldsymbol{\psi}(k))$ is a validity function and $\hat{y}_i(k)$ is the output of the *i*-th model. The network form of Equation 2.6 is shown in Figure 2.

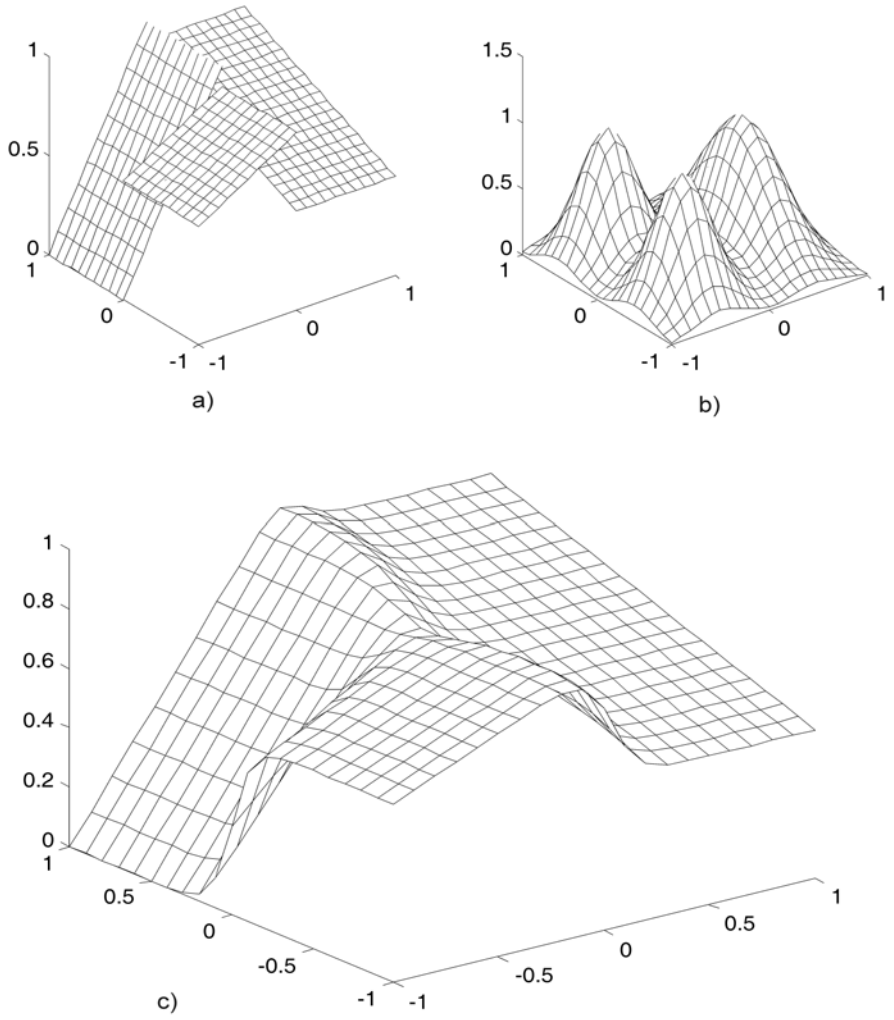


Figure 3 The nonlinear input/output approximation (c) is obtained by combining three linear models (a) with validity functions (b)

The modelling performance of the LMN is depicted in Figure 3, where three local models are combined with three two-dimensional validity functions to produce nonlinear approximation. The assumption for the local modelling approach is that the modelled plant has to undergo significant changes in operating conditions as it moves in the operating space. Introducing the simpler models can reduce the complexity of the nonlinear system. For example, local state-space and ARMAX models can be formed using localised perturbation signals and then blended to produce global nonlinear state-space and NARMAX (nonlinear ARMAX) representations.

2.5 Validity functions

The blending of local models is calculated using the weighting or validity functions. Although any function with locally limited activation might be applied as a validity function, a common choice for this function takes the form of Gaussian. Other popular validity functions as B-splines or multiquadratic functions have been proposed.

Gaussian basis functions are the most common choices for weighting the outputs of local models. The Gauss function for *j*-th model is given by

$$\tilde{\rho}_j(\psi) = \exp\left(-\frac{1}{2}\left(\psi - c_j\right)^T \sigma_j^{-2} \left(\psi - c_j\right) \right) \tag{2.6}$$

where parameters c_j, σ_j^2, define the Gaussian centre and width, respectively and the scheduling variable can be a system state or any system variable. The centre point defines where the model is most active. The number of centres and widths depends on the number of scheduling variable. For one scheduling variable the weighting function is a typical bell-shaped curve.

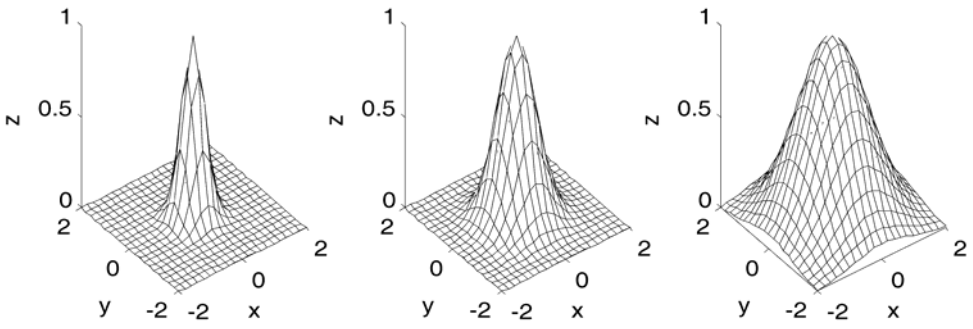


Figure 4 Effect of varying width, σ^2 , for 2-dimensional Gaussian validity function

Normalisation of the basis function is often motivated by the desire to achieve a partition of unity. By partition of unity it is meant that at any point in the input space sum of all normalised basis functions equals unity. i.e.

$$\sum_{j=1}^{M} \rho_j(\boldsymbol{\psi}) = 1 \tag{2.7}$$

Validity function represents the partition of the input space in the local model network structure. The normalised form of the validity function is denoted by $\rho_j(\boldsymbol{\psi})$, for the basis functions associated with local model *j*

$$\rho_j(\psi) = \frac{\tilde{\rho}_j(\psi)}{\sum_{j=1}^{M} \tilde{\rho}_j(\psi)} \tag{2.8}$$

However, normalization also leads to a number of other important side-effects that have consequences for the resulting network [33].

2.6 Side-effects of the normalization of validity functions

The normalized validity functions are often homogenous validity functions with different parameters. Once normalized, the shape of the validity function usually differs from the un-normalized basis function. The shape of the validity function is not only influenced by its width but also by the proximity of other validity functions. It can be seen from Equation 2.9 that each validity function is a function of all the original validity functions. Therefore, a change in the parameters of one validity function influences all other normalized validity functions. Other problem connected with the normalization of validity function is reactivation and shift in maxima. If centres are not uniformly distributed or the validity functions differ in widths, the maximum of the basis function may no longer be at its centre. Furthermore, the validity function can become multi-modal, i.e. it increases as the distance from the original centre increases, instead of continuously decreasing.

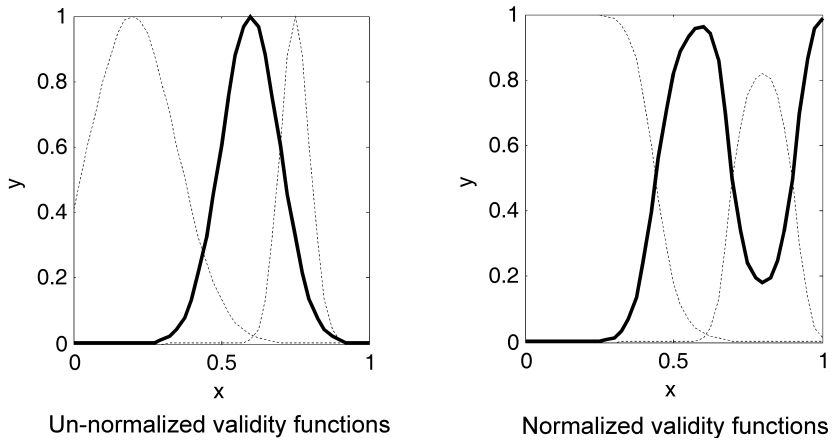


Figure 5 Reactivation of the validity functions for scalar scheduling variable x

In general, assuming monotonically decreasing basis functions, the reactivation point is the point at which the distance metric for the first basis function, d_1, is no longer smaller than that of the nearest basis function, d_2. For the i-th Gaussian function, the distance metric is given by:

$$d_i = \left(\frac{\psi_i - c_i}{\sigma} \right)^2 \tag{2.9}$$

For two neighbouring Gaussian validity functions, the condition for reactivation can be stated as:

$$\frac{\sigma_2}{\sigma_1} < \frac{|\psi - c_1|}{|\psi - c_2|} \tag{2.10}$$

This condition can be regarded as constraint for choosing the centres and widths of the validity functions. Alternatively, to ensure that there will be no reactivation, uniformly wide basis should be used.

B-splines are an alternative choice for the validity functions. The advantage of B-splines over the Gauss functions is their inherent normalization. Therefore, there are no side-effects associated with the normalization. Unfortunately, B-splines are subject to curse of dimensionality, limiting their usefulness for low dimensional problems.

2.7 Local Models

The dynamics of an individual model is of particular interest as it provides basis for the design of local controllers. One of the advantages with local modelling is that the structure of the local models does not need to be very complex. Usually simple linear models are sufficient. However, the local models *f* in Equation (2.2) can be of any form; they can be linear or nonlinear; have state-space or input-output description, or be discrete or continuous-time. The models can be even of different character, i.e. parametric models for operation conditions where description is available *a priori* and neural network where there is lack of physical description. However, this heterogenous LM network would require different optimization techniques and therefore the same type of local models throughout the LMN structure is used.

There is an obvious trade-off between the number of operating regimes and complexity of the local model. If there were only one model for the entire operating space, it would be very complex and it would be the global model. On the other hand, if the operating space were decomposed into numerous small operating regions, the complexity of the local models would be much smaller. If the local model were described only by a constant, LMN structure would be identical to RBF network. The discrete-time nonlinear system is considered to have the general form

$$y(k)=f(y(k-1),..y(k-n_y),u(k-d-1)...u(k-d-n_u),e(k-d-1)....e(k-d-n_e))+e(k) \quad (2.11)$$

Here, $y(k)$ is the system output, $u(k)$ is the input, $e(k)$ is a zero-mean disturbance term and d represents the time-delay. This type of model is known as the NARMAX (Nonlinear ARMAX) model and has been studied widely in nonlinear system identification. If we define the data vector as

$$\boldsymbol{\varphi}(k-1)=[-y(k-1),....,-y(k-n_a),u(k-1),...u(k-n_b),e(k-1),...,e(k-n_c)]^T \quad (2.12)$$

then the system can be rewritten as

$$y(k)=\boldsymbol{\theta}^T\boldsymbol{\varphi}(k-1)+e(k) \quad (2.13)$$

where the ^{T} denotes the transpose operator and $\boldsymbol{\theta}$ is a vector of parameters defined as

$$\boldsymbol{\theta} = \left[a_1, ..., a_{n_a}, b_1, ..., b_{n_b}, c_1, ..., c_{n_c} \right]^T \tag{2.14}$$

The aim in empirical modelling is to find a parameterized structure which emulates the nonlinear function *f*.

Using the Equations (2.13) and (2.14) the linear system can be written in the form

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})} u(k) + \frac{C(z^{-1})}{A(z^{-1})} e(k) \tag{2.15}$$

Here, the z^{-1} is a unit delay operator and the polynomial *A, B, C* are

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + ... + a_{na} z^{-na} \\ B(z^{-1}) &= 1 + b_1 z^{-1} + ... + b_{nb} z^{-nb} \\ C(z^{-1}) &= 1 + c_1 z^{-1} + ... + c_{nc} z^{-nc} \end{aligned} \tag{2.16}$$

An obvious benefit of using local linear models is that their parameters can be identified using standard optimization techniques, once the weighting functions have been defined. Rather than globally learning the local model parameters, it is sometimes better to train each local model individually using locally-weighted, least-squares regression and employing only the training data local to the model.

2.8 Affine modelling

Mathematically, off-equilibrium linearization leads to local affine models, which have an extra degree of freedom. i.e., the bias term to make the models more flexible, so they can be shifted upwards and downwards in the operating space. The bias term can significantly improve the modelling accuracy of the LLM. However, due to the bias term the affine models do not possess the superposition property fundamental to the linear systems. Thus, there is a lack of continuity with established linear theory. The inhomogeneous term can become large and significantly influnce the solution, while the change of parameters *A* and *B* only have minor influence on the local model accuracy. Therefore, the bias cannot be simply regarded as a small approximation error or disturbance. With local linear models , one can achieve accurate approximation of either the linearized dynamics or the trend, but not both simultaneously. With the

affine model structure there are no such limitations [34]. Since strictly linear models have a common point in the origin (Figure 6), local affine models clearly result in an accurate approximation with a valid interpretation as a local linearization.

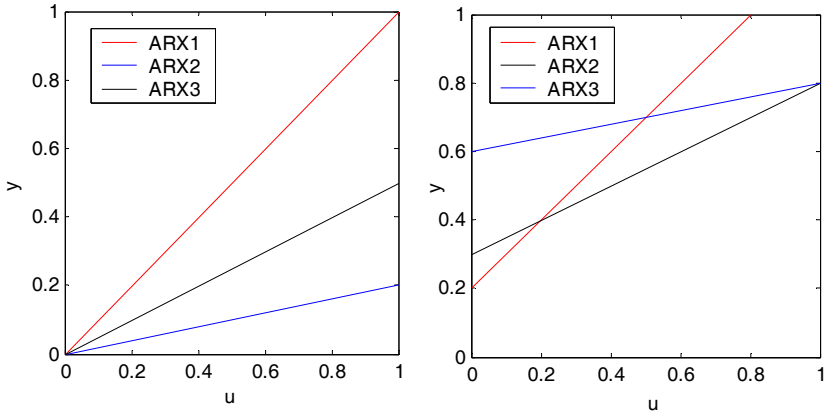


Figure 6 Strictly linear and affine ARX models

2.9 Fuzzy-based local modelling

The use of weighting functions for partitioning the system's operating space into several operating regimes is not restricted to local model networks. A similar approach can be found in literature in the guise of the Takagi-Sugeno (TS) fuzzy models. In fact, for specific conditions, the LM network and TS fuzzy have equivalent functional behaviour. The LMN network is functionally equivalent to the TS fuzzy model if the following conditions are satisfied:

1. The number of local models in the LM network is equal to the number of *if-then* rules.
2. The membership basis functions within each rule are chosen as Gaussian functions.
3. The operator chosen to compute each rule's firing strength is the product.
4. Both the LM network and the TS model use the same method to derive the overall outputs, i.e. either normalised or un-normalised weights.

2.10 Concluding remarks

The main features of the local model network have been described in this chapter. It has been shown that LMN can be viewed as generalization of the Radial Basis Function network where output constants at each neuron are replaced by a local model. In the research literature the local models are often linear and the validity functions take form of Gaussian basis function. To provide partition of unity the validity function are normalized which can cause side-effects such as shift in maxima or loss of local support. These side effects degrade the transparency and interpretability of the network. These side-effects can be avoided by using B-splines, however due to the curse of dimensionality; they are limited to the low dimensional problems. The link between the LM network and the fuzzy-based Takagi-Sugeno model has been also shortly outlined in this chapter. Under certain conditions, the two global models are functionally equivalent. Thus, research analysis for both nonlinear representations is interchangeable. The advantage of the LMNs is that it does not suffer from the "stability-plasticity dilemma" which is a common design problem with adaptive control where the algorithm is adapting to the new operating region, it is also forgetting the previous regions.

Chapter 3 NONLINEAR SYSTEM MODELLING USING LOCAL MODEL NETWORKS

Modelling nonlinear dynamic systems from the observed data and prior knowledge is an important area of science and engineering. Training is a key issue in the application of the Local Model Networks because there is the added complexity of having to determine the number and the structure of sub-models as well as the parameters of the validity functions in addition to parameter identification,. There are several methods for modelling a nonlinear system and the choice of a particular modelling method depends on the aim of modelling. If the aim of modelling is control design then the identification technique should lead to simple, transparent and mathematically tractable models. In many applications it is necessary to combine the information obtained from the numerical data with heuristic knowledge. Another major requirement for nonlinear system modelling algorithm is the universality in the sense that a wide class of structurally different systems can be described. The described architecture of local model networks is capable of fulfilling these requirements and can be applied to tasks where high degree of flexibility is required. Construction of models of such structure involves a linear estimation of the regression models and nonlinear optimization of the parameters of the validity functions. A general approach to parameter estimation is to minimize some criterion that measures the difference between the output of the model and the observation data. This is called global learning. If the parameters of each model are identified separately using the weighted subset of data the approach is called local modelling.

3.1 *Divide and Conquer* Strategy

Divide and Conquer approaches have drawn attention in recent years in the area of controlling nonlinear plants over a wide range of the operating conditions. These approaches decompose the complex nonlinear problem in a number of simpler problems, each associated with restricted operating region. This results in transparent representation of the control system with the opportunity to use the conventional linear control design techniques. The Divide-and-Conquer strategies can be divided into two groups: analytical approaches and the learning approaches. While the first one requires the analytical form of description of the system to be controlled, the latter is based on the input-output observations data.

Analytical approaches

- Linearization
- Gain scheduling
- Feedback linearization

Learning approaches

- Modular architecture
- Local learning

When no analytical model is available but the assumption of linearity is reasonable, linear system identification methods are used to estimate the system's parameters. Linear techniques have been also employed for the nonlinear systems using the adaptive algorithms. Here the on-line identification algorithm provides the linear approximation of the nonlinear system in order to track variations in the dynamics. The idea of learning techniques is to approximate the unknown relation between state of the system and control actions by using nonlinear optimization techniques. The divide-and-conquer idea has taken two different forms: the global learning and the local learning. Although the global learning uses a combination of local models, their identification is still performed on the basis of the whole dataset. Although the simplification can be made by using the local linear models, the problem is still nonlinear and requires the same procedures used for generic global models.

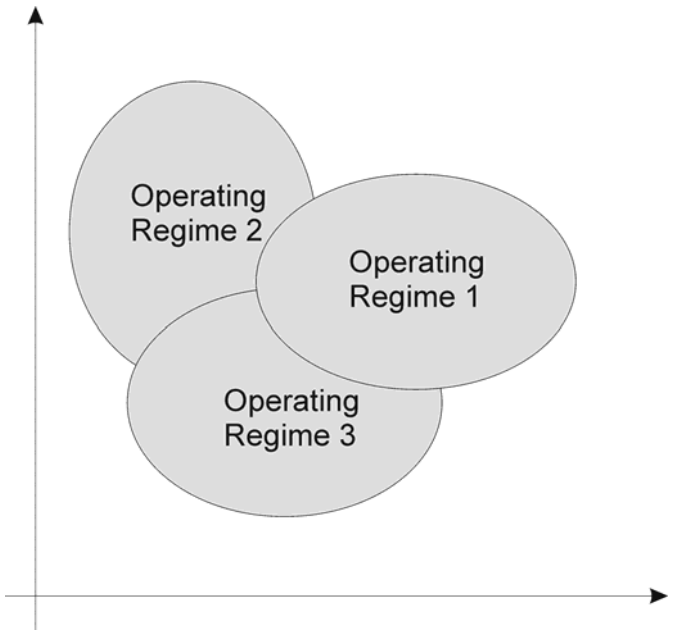


Figure 7 The operating range of complex systems is decomposed into a number of operating regimes (the axis represent two scheduling variables)

3.2 The modelling process

The divide-and-conquer strategy helps to improve the techniques for the design of models of nonlinear systems with the aid of computationally data-driven techniques. The modelling process involves integrating the knowledge about the system with the experimental data. The typical sources of information of the system could be:

- Experimental data, such as responses to perturbations
- A possibly incomplete nonlinear model that may be too simple or too complicated
- Qualitative knowledge, i.e. behaviours or engineer's heuristics

The model can only represent certain aspects of the system, so it is necessary to know the purpose of the model in order to decide which aspects should model capture.

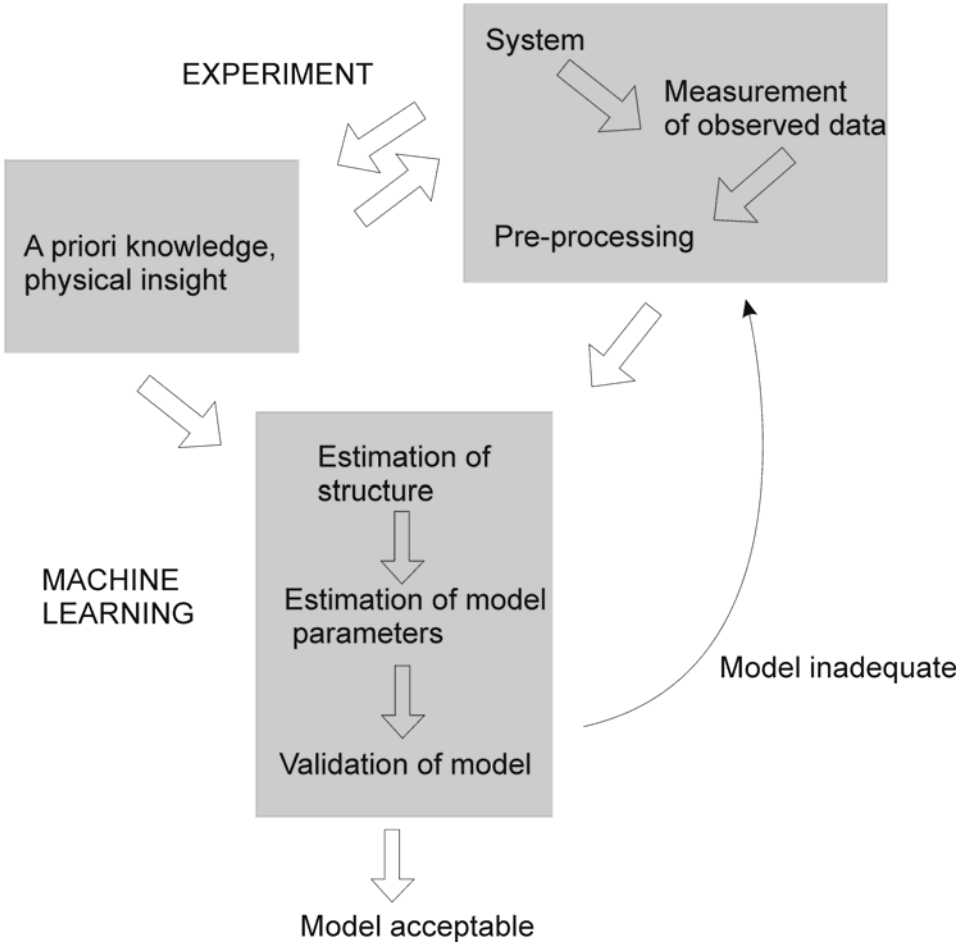


Figure 8 Engineering approach to model development

The abstract modelling cycle (Figure 8), which covers a number of further tasks that are essential in modelling:

- Experiment design and data acquisition
- Raw data processing and analysis
- Analysis of *a priori* knowledge. Physical laws and available models
- Structure and parameter optimization
- Model reduction and simplification
- Model validation, analysis and interpretation

The experimental data is used for the parameter and structure identification so it is necessary that the data covers the important aspects of the system. The input signal should exhibit enough amplitude and appropriate frequency range in order to excite all interesting modes of the plant, however these requirements are often in disagreement with the industrial practice. Once the experiments are designed, the actual input and output sequences have to be collected. In many processes it is important to have accurate models in stable areas than less accurate models throughout the whole operating range, as the system usually spends most of the time in the stable area. It is also necessary to have appropriate amount of identification data in the regions where the system is most complex or the control is most critical. Lack of data from a certain operating region can explain why the related local model parameters are not accurately identified. As it can be seen, the choice of appropriate data for structure and parameter identification is a crucial part of the modelling and requires more consideration then subsequent machine learning [34].

3.3 Structural identification

Although *a priori* knowledge is important to define the model structure, in some cases the system complexity is not well understood for the model structure to be specified in advance. So it is often necessary to adapt the structure based on the information in the training data. Optimization of the model structure M is, however, a difficult non-convex optimization problem. The goal of the structure identification procedure is to relate the density and the size of operating regimes to the complexity of the system. The desirable features of the identification algorithm:

- Convergence – as the number of training points increases the algorithm should provide more accurate model of the system being modelled
- Parsimony – the model structure should be the simplest possible to achieve the required accuracy
- Robustness – the model structure produced should be robust to the noisy data

- Interpretability – the model structure should be as interpretable as possible given the local models, their validity functions and available data

The techniques for the optimization of positions and dimensions of the local regions fall into several classes:

- **Fixed Selection**

  In this approach the centres are selected randomly from the input data or distributed uniformly [36]. The widths of operating regions are calculated to some thumb rules based on *a priori* information. If the complexity of the problem is unknown a large number model is necessary for fine approximation of the nonlinear system. This is a rather bad clustering solution because even a small region can be highly nonlinear.

- **Self-organizing and clustering**

  In this approach the centres of the operating regimes are trained in an unsupervised learning fashion. Abonyi *et al*, in [38] and [39] used Expectation Maximization algorithm (EM) to identify simultaneously and directly the operating regimes and the parameters of the local models. The disadvantage of these algorithms is that the local models are clustered according to the density of the data, not according to the complexity of the problem.

- **Non-optimal construction algorithms with heuristic growing strategies**

  These techniques start with a simple structure, e.g. a global linear model, and divide the input space into smaller areas. Examples are local linear model tree (LOLIMOT) [10], Johansen and Foss algorithm [5], algorithm of Aarhus [40] that is trying to find a split point in which to carry out the decomposition of the input space to reduce the prediction error. Kavli in [41] developed an ASMOD algorithm that uses B-Splines to represent general nonlinear and coupled dependencies in multivariable observation data. Jakubek and Kreuth [42] created an iterative construction algorithm that tries to fit the local models to the available data using statistical criteria along with regularization.

- **Splitting and merging**

  These algorithms try to adjust the network complexity according to the complexity of the problem. The algorithm splits an operating region into two, if the behaviour of the model is not satisfactory and two neighboring models are merged together if they have nearly identical parameters to limit the complexity of network.
- **Fine-to-course learning**

  These techniques start with a large number of local models and during training the local model are merged together to get a simple structure [43].

While the comparison between the algorithms of Johansen and Foss and Nelles can be found in [44], the work by McGinnity and Irwin [45] compares the hybrid optimization algorithm by McLoone and Johansen and Foss algorithm.

All the structure identification algorithms are computationally expensive. This problem becomes crucial if a number of models or input dimension of the network becomes large. Therefore, it is always advantageous to consider the maximum possible *a priori* information about the system.

3.4 Heuristic strategies for structure identification

The following part addresses the problem of identifying a model of an unknown non-linear system on the basis of a sequence of N input/output pairs

$$\boldsymbol{D}_N = \big((u(1), y(1)), \ldots\ldots\ldots, (u(N), y(N))\big) \tag{3.1}$$

where $y(k), u(k)$ are the system outputs and input respectively. A global model can be formed

$$\hat{y}(\boldsymbol{\psi}) = \sum_{i=1}^{M} f(\boldsymbol{\theta}_i, \boldsymbol{\varphi})\rho(\boldsymbol{\psi}) \tag{3.2}$$

where $\boldsymbol{\psi}$ is the vector of scheduling variables. The addressed problem is estimation of this function, since the function immediately gives the model equations. The model structure based on modular description of N regimes can be written

$$M_N = \left\{ Z_i, \rho_i, \hat{f}_i \right\} \tag{3.3}$$

where regime Z_i represents subspace of the whole operating space. The modelling problem consists of the following sub-problems:

1. Choose the variables which to characterize the operating regimes with
2. Decompose Operating space Z into regimes and choose local model structure
3. Identify the local model parameters for all the models

An appropriate order for the identified model may be determined by the Akaike information criterion (AIC) [46]. The procedure is to repeat the identification process for different orders and choose the final model by looking for a small AIC value, together with appropriate model dynamics.

The AIC is defined as

$$AIC = \log\left(J\left(1 + 2\frac{d}{N} \right) \right) \tag{3.4}$$

where d is a total number of estimated parameters, N is the length of the data record and J is the loss function for the structure in question.

3.4.1 Johansen and Foss Algorithm

For modelling the nonlinear process the J&F algorithm [5], which incorporates an outer loop for structure optimization and inner loop for parameter identification, can be used. The scheduling variables have to be known beforehand. The J&F algorithm starts with only one model and the weighting function is unity over the whole operating d-dimensional space Z, where d is the number of scheduling variables. Using the least-squares method, the parameters estimation of the only model can be performed. The algorithm then divides the operating space into two parts (Figure 9). Since an infinite number of divisions is possible it is necessary to reduce the number of divisions being investigated. This is done by only allowing the regime to be divided in a direction parallel to an axis of the box *Z1* which defines an operating regime, and at a finite number of points along each axis.

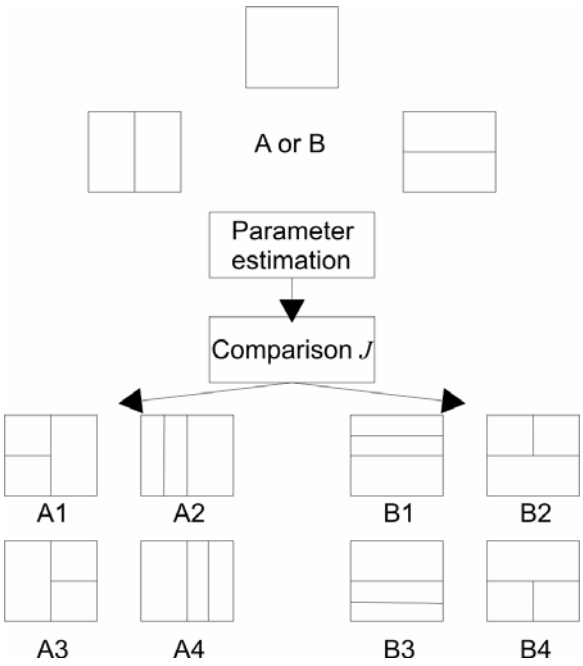


Figure 9 Operating regime decomposition

Thus for a *d*-dimensional box with *s* splitting points, a total of *ds* new decompositions are formed. New parameters of weighting functions are determined by the limits of each working regime

$$c_j = 0.5(z_j^{\max} + z_j^{\min})$$
$$\sigma_j = 0.5\gamma(z_j^{\max} - z_j^{\min}) \tag{3.5}$$

where parameter γ influences the overlapping of Gauss function and $z_j^{\max}$, $z_j^{\min}$ are limits of the *j*-th regime. For small values of γ the functions will not overlap and for large values the transition from one region to another will be smooth. Once the weighting functions parameters are obtained, the parameters of the local models can be estimated by least-squares method (Chapter 3.7) or weighted least-squares method (Chapter 3.8).

For each split the value of cost function is calculated and the parameters of local models are determined. After considering all possible splits, the one with the lowest cost function is then chosen and the procedure is repeated. This process continues until either a maximum number, *M*, of regimes is found or until some pre-specified

modelling cost criterion is satisfied. The construction algorithms can also effectively determine which variables are required to suitably decompose the operating space. If no splits are formed over a particular axis, then the variable associated with that axis can be ignored.

3.4.2 LOLIMOT algorithm

This method splits up the identification procedure into two parts. In the outer loop, the structure of the local model network is optimized by a tree construction algorithm. In a inner loop the parameters of the local models are estimated by local modelling technique. The construction algorithm partitions the input space by orthonormal cuts dividing the worst performance local model along the input axis, which yields the highest improvement [10]. Consequently, the nonlinear model complexity automatically adapts to the complexity of the process.

At first sight, both algorithms appear to be very similar. However, there is an obvious trade-off between computation complexity and model optimality. While the J&F technique tries to achieve an optimal model at the expense of intensive computation, LOLIMOT minimizes the computational effort involved at the risk of producing a sub-optimal model. In general, the J&F algorithm produces a more accurate representation than LOLIMOT but with a considerable increase in computational effort required. Comparison on the computational costs and modelling performances of both techniques can be found in [44].

3.5 Structure optimization via the SOMA algorithm

Self-Organizing Migration algorithm (SOMA) [48] is a genetic algorithm that is based on the competitive-cooperative behaviour of intelligent creatures solving a common problem. Such behaviour of intelligent creatures can be observed anywhere in the real world. A group of wolves or other predators may be a good example. If they

are looking for food, they usually cooperate and compete so that if one member of the group is more successful than the previous best one (e.g. has found more food) then all members change their trajectories towards the new most successful member. The procedure is repeated until all members meet at one food source. In SOMA, wolves are replaced by individuals. They ‘live’ in the optimized model’s hyperspace, looking for the best solution. This kind of behaviour of intelligent individuals allows SOMA to realize very successful searches.

Because SOMA uses the philosophy of competition and cooperation, the variants of SOMA are called strategies. They differ in the way the individuals affect all others. The basic strategy is called 'AllToOne'. Before starting the algorithm, the SOMA parameters such as population size or number and migrations has to be defined. The user must also create the specimen and the cost function that will be optimized. Cost function is a wrapper for the real model and must return a scalar value, which is used as gauge of the position fitness.

SOMA, as well as the other evolutionary algorithms, works on a population of individuals. Each individual represents an actual solution of the given problem. In fact, it is a set of parameters for the cost function, whose optimal setting is being searched. The cost function response (cost value) to the input parameters is associated with each individual. The cost value represents the fitness of the evaluated individual. It does not take part in the evolutionary process itself; it only guides the search process.

For LMN optimization with local ARX models an individual that represents possible solution of the optimization problem have the following structure:

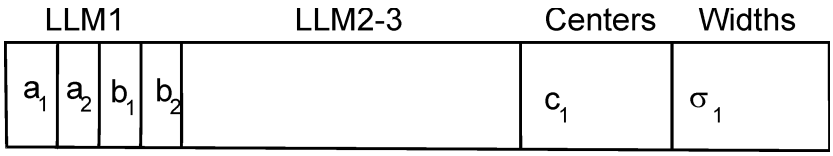


Figure 10 Individual representing possible solution of the optimization problem

Usage of all the model parameters for optimization leads to the same problems as in the global learning technique since each local model influences all the other models. Possible solution is combination of SOMA optimization algorithm for optimization of

the validity function parameters combined with the least-squared method for local model parameters estimation.

In the SOMA optimization process it is easy to penalize the LMNs with unstable local models or with the reactivation of the validity function by simply adding a constant to the value of modelling criterion to prevent these LMNs to create possible global optimum of the optimization problem.

3.6 Parameter Estimation in Local Model Network Structure

The problem of parameter estimation for systems which are linear in parameters is reasonably well understood, with variety of efficient optimization techniques to optimize the parameters $\boldsymbol{\theta}$ of the local models. Parameter optimization for a given model structure finds the optimal costs

$$J^*(\boldsymbol{M},\boldsymbol{D}) = \min J(\boldsymbol{\theta},\boldsymbol{M},\boldsymbol{D}) \tag{3.1}$$

where $\boldsymbol{M}$ is a given structure of the network $\boldsymbol{M} = (\boldsymbol{c},\boldsymbol{\sigma},M_1...M_{n_M})$ (i.e. centers and widths of the validity functions and local models structure) and D is the training data set $\boldsymbol{D} = (\varphi(k-1), y(k), k = 1..N)$.

3.7 Global learning

If we assume models linear in parameters and fixed parameters of validity function, the learning problem is the application of the least-squares method to obtain the parameters $\widehat{\boldsymbol{\theta}}$. Using the matrix form we obtain the following regression model

$$\boldsymbol{y} = \boldsymbol{\theta}^T\boldsymbol{\Phi} + \boldsymbol{\varepsilon} \tag{3.2}$$

where the matrix $\boldsymbol{\Phi}$ contains the rows defined by the term

$$\boldsymbol{\Phi}_k = \left[\rho_1(\boldsymbol{\psi}(k))\boldsymbol{\varphi}(k), ..., \rho_M(\boldsymbol{\psi}(k))\boldsymbol{\varphi}(k)\right] \tag{3.3}$$

where $\boldsymbol{\varphi}(k)$ is a regression data vector. In Equation (3.3) the values of the validity functions are fixed either for one individual of the SOMA algorithm or one possible division point of the Johansen and Foss algorithm.

Matrix $\boldsymbol{\Phi}$, regression vector $\boldsymbol{\varphi}(k)$, output data vector $\boldsymbol{y}$, error vector $\boldsymbol{\varepsilon}$ and parameters vector $\hat{\boldsymbol{\theta}}$ are defined as follows:

$$\begin{aligned}
&\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \vdots \\ \boldsymbol{\Phi}_N \end{bmatrix} \\
&\boldsymbol{\varphi}(k) = \left[-y(k-1), ..., -y(k-na), u(k-1), ..., u(k-nb)\right] \\
&\boldsymbol{y} = \left[y_1, ..., y_N\right] \\
&\boldsymbol{\varepsilon} = \left[\varepsilon_1, ..., \varepsilon_N\right] \\
&\hat{\boldsymbol{\theta}} = [\hat{\boldsymbol{\theta}}_1, \ldots, \hat{\boldsymbol{\theta}}_M]^T \\
&\hat{\boldsymbol{\theta}}_j = \left[a^j{}_1, ..., a^j{}_{na}, b^j{}_1, ..., b^j{}_{nb}\right]^T
\end{aligned} \tag{3.4}$$

Criterion for the least squares estimation is given by

$$J(\hat{\boldsymbol{\theta}}) = \frac{1}{N}\left[\boldsymbol{y} - \hat{\boldsymbol{\theta}}^T\boldsymbol{\Phi}\right]^T\left[\boldsymbol{y} - \hat{\boldsymbol{\theta}}^T\boldsymbol{\Phi}\right] \tag{3.5}$$

where N stands for the size of measured data and parameter estimates can be obtained using the equation

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\Phi}^+\boldsymbol{y} = (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\boldsymbol{y} \tag{3.6}$$

where Moore-Penroseho pseudoinverse $\boldsymbol{\Phi}^+$ [35], can be calculated using the singular decomposition. Using the singular decomposition the matrix $\boldsymbol{\Phi}$ can be divided into terms so that:

$$\begin{aligned}
\boldsymbol{\Phi} &= \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^T \\
\boldsymbol{\Phi}^+ &= \boldsymbol{V}\boldsymbol{S}^+\boldsymbol{U}^T
\end{aligned} \tag{3.7}$$

The numerical algorithm for the calculation of the pseudoinverse is called Singular Value Decomposition SVD. Once the singular values have been zeroed, the parameters of the local models can be computed.

$$\hat{\boldsymbol{\theta}} = \boldsymbol{V}\boldsymbol{S}^+\boldsymbol{U}^T\boldsymbol{y} \tag{3.8}$$

3.8 Local Learning

Global learning is based on the assumption that parameters of all the local models are estimated in one step. In general, the global learning is more accurate than local learning. In Murray-Smith and Johansen [4] showed that global learning does not guarantee to produce local models that will be close approximation to local linearization of the system. Global learning can also become computationally if a large number of models or data are used. The other choice which does not posses the aforementioned disadvantages is to estimate independently parameters of each model.

Parameters of the local models are estimated using the weighted criterion for *i*-th model

$$J_j(\hat{\boldsymbol{\theta}}_j) = \frac{1}{N}\left[\boldsymbol{y} - \hat{\boldsymbol{\theta}}_j^T \boldsymbol{\Omega}\right]^T \boldsymbol{Q}_j \left[\boldsymbol{y} - \hat{\boldsymbol{\theta}}_j^T \boldsymbol{\Omega}\right] \tag{3.9}$$

where $j = 1,...M$. $\boldsymbol{Q}_j$ is an *N* x *N* diagonal weight matrix, where diagonal elements of the matrix are used to weight the importance of the different samples in the training set on *i*-th local model. The matrix of measured data $\boldsymbol{\Omega}$ is defined as

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\varphi}(1) \\ . \\ . \\ \boldsymbol{\varphi}(N) \end{bmatrix} \tag{3.10}$$

The importance of the samples to *j*-th model is given directly by the unnormalized individual validity function that are fixed either for one individual of the SOMA algorithm or one possible division point of the Johansen and Foss algorithm.

$$\boldsymbol{Q}_j = diag(\tilde{\rho}_j(\boldsymbol{\psi}(1)),..., \tilde{\rho}_j(\boldsymbol{\psi}(N))) \tag{3.11}$$

Parameters $\hat{\boldsymbol{\theta}}_j, \boldsymbol{y}$ in the Equation (3.9) remain the same as in the previous chapter. The locally weighted estimates of parameters of a local model $\hat{\boldsymbol{\theta}}_j$ are given by minimum of the criterion J_j .

$$\hat{\boldsymbol{\theta}}_j = \left(\boldsymbol{\Omega}^T \boldsymbol{Q}_j \boldsymbol{\Omega}\right)^{-1} \boldsymbol{\Omega}^T \boldsymbol{Q}_j \boldsymbol{y} \tag{3.12}$$

3.9 Incorporating *a priori* knowledge

A major advantage of the local model networks is that they are not only useful architectures for general learning tasks, but that it is relatively easy to introduce the *a priori* knowledge about a particular problem. *A priori* information is initial knowledge about the system or a problem in question. As can be seen from Figure 8 *a priori* knowledge plays an important role at each stage of the modelling process. *A priori* knowledge includes goals of the problem, characteristics of the system, its parameters and effect of the environment (disturbances and noise). The most general form of information is the expected dynamic order of the models, the form of the models (e.g. ARX models) and the sampling period. In many cases, there will not be sufficient data to train the model throughout the input space, especially outside the areas of normal operation. This can be overcome by fixing *a priori* models to the areas where system is well understood and applying the learning techniques only where data is available and reliable. A further option, for cases where a complex and too complicated model exists and is valid for certain operating regimes, is to pre-set the fixed local models obtained through the linearization of the nonlinear model. The linearization of the nonlinear model can also be used for comparison with the model obtained from the experimental data.

The problem of choosing the local model dynamic order and the sampling period is not a simple task. The problem of order selection and sampling period is widely discussed in the identification literature [49].

Determining model parameters from a finite set of observation is an ill-posed problem, since a unique model may not exist or it my not depend on the observations. Even if the data are not corrupted by noise, the model can exhibit random behaviour at an operating point that is not exactly captured by the observation data set. It is often desirable to employ all the available prior knowledge and observation data to obtain a better conditioned problem. This will generally lead to a better model. The available prior knowledge used through optimization can have several forms: smoothness of the model behaviour, partially or completely known local models, constraints on the

validity function, constraints on the parameters (stability, process gain, settling time) and empirical data measured through experiments.

Knowledge about the process gain, stability and settling time could be translated into the form of inequality constraints. Thus optimization in the form of quadratic programming (QP) can be used to obtain model parameters, instead of conventional least-squared method as described in the following section.

If the system to be identified is assumed stable there exist several limits on the parameters of local models. For the system of the second order with the denominator given by

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{3.13}$$

the following stability margins for the parameters a_1, a_2 can be inroduced if the system is stable

$$\begin{aligned} a_2 &\le 1 \\ -a_2 + a_1 &\le 1 \\ -a_1 - a_2 &\le 1 \end{aligned} \tag{3.14}$$

which can be translated as the inequality conditions for QP

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix} \le \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{3.15}$$

If the interval of the static gain or at least the sign of the gain is known beforehand the following inequality condition has to be satisfied:

$$K_{\min} \le \frac{\sum_{i=1}^{n_b} b_i}{1 + \sum_{i=1}^{n_a} a_i} \le K_{\max} \tag{3.16}$$

So the linear inequality constraints are given

$$K_{\min}\left(1+\sum_{i=1}^{n_a} a_i\right)-\sum_{i=1}^{n_b} b_i \le 0,$$
$$-K_{\max}\left(1+\sum_{i=1}^{n_a} a_i\right)+\sum_{i=1}^{n_b} b_i \le 0 \tag{3.17}$$

If the global modelling approach is employed in the training algorithm and following regression model is used

$$\boldsymbol{y} = \boldsymbol{\Phi}^T \boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{3.18}$$

with affine ARX models of the second-order, the regression matrix and vector of outputs are defined:

$$\boldsymbol{y} = \begin{bmatrix} y(3) \\ y(4) \\ : \\ y(N) \end{bmatrix} \tag{3.19}$$

$$\boldsymbol{\Phi}_k^T = \left[\rho_1(\boldsymbol{\psi}(k))\boldsymbol{\varphi}(k), ..., \rho_M(\boldsymbol{\psi}(k))\boldsymbol{\varphi}(k)\right] \tag{3.20}$$

The least-squares method can be solved by

$$\hat{\boldsymbol{\theta}} = (\boldsymbol{\Phi}^T \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^T \boldsymbol{y} \tag{3.21}$$

The constrained optimization problem can be formulated as:

$$\min_{\hat{\theta}} \left\{ \frac{1}{2} \hat{\boldsymbol{\theta}}^T \boldsymbol{H} \hat{\boldsymbol{\theta}} + \boldsymbol{c}^T \hat{\boldsymbol{\theta}} \right\} \tag{3.22}$$

with

$$\boldsymbol{H} = 2\boldsymbol{\Phi}^T \boldsymbol{\Phi}, \quad \boldsymbol{c} = -2\boldsymbol{\Phi}^T \boldsymbol{y} \tag{3.23}$$

and the constraints

$$\boldsymbol{A}_{inq} \hat{\boldsymbol{\theta}} \le \boldsymbol{b} \tag{3.24}$$

By using the constraints during the training process, more accurate model with improved interpretability can be identified using the input-output data [50].

3.10 Validation

Practically it is impossible to develop a model that would completely describe the true system. There will be always some features in the true system, which the model cannot describe. At this stage the quality of the mode is evaluated by analyzing how well it captures the data. Usually, validation is performed by combination of statistical measures that evaluate the generalization capability of the mode, and qualitative criteria, focus on establishing how the model relates to *a priori* knowledge, how it easy to be used and interpreted. In literature there are several methods for model validation, see [49] for a thorough discussion. The most common technique is to compare the predictions generated by the model with the measured data.

When validating a model a *cross validation* method is often used. This means that a model is validated using new data. For this purpose the data set is divided into two parts where one set is used for identification purposes and one for validation.

3.11 Concluding Remarks

At the beginning of the chapter the basics of divide-and-conquer strategy and identification from experimental data are given. An important aspect when creating a learning algorithm for local model structure is the trade-off between achieving a good global fit and good local representation. Global learning methods are computationally more expensive and obtained models can not be interpreted as valid linearizations of the underlying nonlinear system because each model is influenced by all the other models. When identifying the local models by minimizing locally weighted prediction error criteria, the local models have locally valid interpretations as linearizations. On the other hand, the global prediction performance is typically inferior to what can be achieved by global optimization. Analysis and comparison of local and global learning method can be found in [4]. A learning algorithm for a local model network has to perform two tasks: identification of the network structure and estimation of local model parameters. Structure identification comprises of the determination of the number and

distribution and shape of the operating regions and the parameters of the weighting functions. Description of various learning techniques for training of local model networks has been given in the previous sub-sections. Constructive techniques which gradually increase the number of models to achieve better model representation have several advantages. Firstly, the main features of a process are captured first, then details. The overfitting protection of the constructive algorithm also limits overtraining. Since the number of divisions is restricted in these algorithms, better global fit can be achieved via optimization technique that utilizes the evolutionary algorithms. SOMA algorithm for the construction of the local model network has been proposed. The approach optimizes either all the local model parameters or only the parameters of the validity functions and constraints that ensure transparency and local interpretability can be easily integrated. The use of constraints is useful to identify interpretable local models, especially when the assumption for excitation is not strongly satisfied. The modelling performances of the training algorithms approaches have been evaluated using simulated data from pH neutralization process in the experimental part of this thesis.

Chapter 4 CONTROLLER DESIGN BASED ON LOCAL LINEAR MODEL DESCRIPTION

The local model network control structure utilizes, in most cases, two local model networks. One network is responsible for identification, while the other servers as a controller. In an adaptive scheme the identification network is updated at each sampling interval.

The multiple model controller design method is characterized by three steps

1. the plant is modelled by local model network composed of M local linear models
2. a feedback controller is designed for each local model in order to guarantee local stability and robustness
3. global controller is obtained by combining the local controllers via the validity functions

This approach seems quite similar to the Gain scheduling approach but there are two main differences:

1. In the LMN scheme the local models are not linearized about the equilibrium points but about generic operating points
2. Unlike the Gain scheduling, the LMN approach requires no analytical description of the system

Basically, there are two ways to design controllers for local model structures, the linearization based and the local model-based approach. For linearization-based approach the local model network is linearized at the current operating point and linear

controller is designed. The linearization of the LM network is very simple due to the structure of the model. This approach is used for the controller design in the next sections. In the second approach a local controller is designed for each local model and the control output is then calculated as an interpolation of the local controller according to the current operating point.

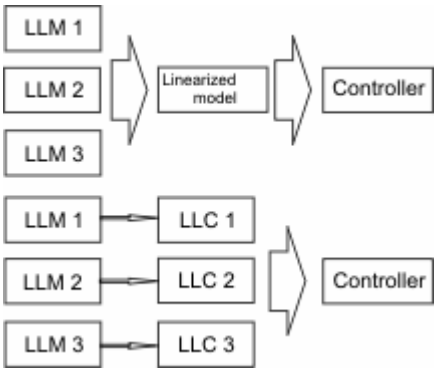


Figure 11 Controller design using linearization and local models (LLM – local linear model, LLC local linear controller)

Only the SISO systems are considered within this chapter, however, extension for the MIMO case can be easily derived for predictive controller.

4.1 Local Controller Networks

Once the LMN is constructed, local controller network, the control version of LMN is defined in turn. In general, the global control signal is determined by

$$u(k) = \sum_{i=1}^{M} C_i(\boldsymbol{\varphi}^c(k), \rho_i(\boldsymbol{\psi}(k))) \tag{3.25}$$

where C_i denotes the local controller for a local model *i*. The local controllers are blended using the same validity functions ρ_i, which are used in LM network. The controller information vector $\boldsymbol{\varphi}^c$ consists of past controller outputs, current and past plant outputs and the current and past values of reference signal. There are two

different possibilities of handling the states of the controller. Either the local controllers may share common states or each controller can have its own states.

4.2 Model Predictive Control

Model-based Predictive control is becoming widely used in industry due to its ability to handle difficult control problems such as multi-input multi-output processes or constraints in the system variables. Model Predictive Control (MPC) refers to a class of algorithms that compute a sequence of manipulated variable adjustments in order to optimize the future behaviour of the plant. Camacho and Bordons in [51] provide practical aspects of the most commonly used MPC strategies and also an overview of commercially available MPC technology. MPC algorithms possess common elements, in which different options for each element give different algorithm.

- Most MPC technologies are based on linear input-output models (transfer functions, step response models). These models are usually identified from the plant input-output data. The accuracy of the identified model is crucial for the performance of the MPC algorithm.
- At each sampling point the inputs are determined from optimization of a quadratic performance objective over finite prediction horizon.
- Future outputs are specified by set-points or reference trajectories.
- Input and output constraints, which can be hard or soft, are included in the optimization process.

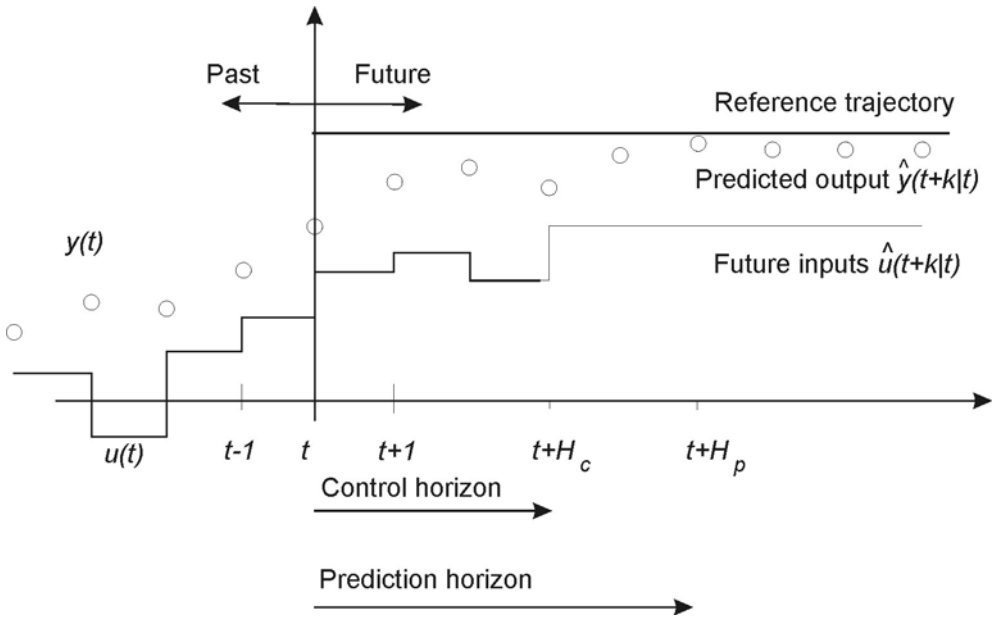


Figure 12 The receding horizon strategy, the basic idea of predictive control

The basic idea of the model predictive control (*Figure 12*) is to determine the control action *u(k)* at time $t = kT$, by using the sampling period time T and solving a finite-horizon optimization problem over a time interval of $t \in [kT, (k+N)T]$. At the new time instant a new control *u(k+1)* is found by solving a new optimization problem for the next time interval. The one-step prediction of the nonlinear system is given

$$\hat{y}(k+1) = \hat{f}\left(\left[y(k), ..., y(k-n_y+1), u(k), ..., u(k-n_u+1)\right]\right), \qquad (4.1)$$

where $\hat{f}$ is an approximation of the nonlinear system. Moreover, the input and output vectors

$$\hat{y}(k+i) \in Y, u(k+j) \in U, i = 1,.., H_p, j = 1,.., H_c \qquad (4.2)$$

u(k),y(k) must lie within the boundaries given by Y and U which are compact sets R^m and R^r respectively. The control is determined by minimization of the quadratic objective function defined as

$$J = \sum_{i=1}^{H_p} e^T(k+i)e(k+i) + \sum_{j=1}^{H_c} \lambda(j)\Delta u^T(k+j) \qquad (4.3)$$

where the vector $e(k+i) = w(k+i) - \hat{y}(k+i)$ represents the error between the desired output and predicted output at time $t = (k+i)T$. H_p, H_c are prediction and control horizons respectively. Δu is defined as an incremental change $\Delta u(k) = u(k) - u(k-1)$ since it is the change of the input signal that is unwanted. The coefficient λ is usually a constant or an exponential sequence that penalizes the differences of future efforts. Since the predictive control strategy is based on the receding horizon, so that at each instant the horizon is displaced towards the future, which involves application of the first control signal of the sequence solved at each sampling time.

Although only the first element of the vector $u(k),...,u(k+H_c-1)$ is used for control, the other elements can be effectively used to initiate the optimization procedure at the step $t = (k+1)T$.

4.3 Nonlinear Model Predictive Control

In general, industrial processes are nonlinear, but most MPC applications are based on the use of linear models. The first reason is that identification of a linear model is relatively simple and the linear models are sufficient when the plant is operating in the neighbourhood of the operating point. Secondly, the use of linear models with quadratic objective function gives rise to a convex problem which can be solved by Quadratic Programming. Thus two major issues limit its application to nonlinear systems. Firstly, the assumption for the predictive control is a quite accurate model of the system. However, complex systems are often connected with nonlinearities, wide operating range or uncertainty. The second is that a nonlinear non-convex optimization problem must be solved for each sampling period with very computationally demanding algorithms. A wide variety of models has been used for nonlinear model predictive algorithms.

These include:

- Nonlinear differential equations
- Hammerstein [52], Wiener, Volterra [53]and Laguerre models [54]

- NARMAX models
- Neural networks [14],[55],[56],[57]
- Fuzzy models

The local linear model structure offers several advantages in the model predictive control scheme. The LLM network provides good prediction of the future trajectory of the system but also the parameters of the plant at each operating point. If the local models are of the affine ARX form

$$f_i(\psi) = a_0 + a_1^i y(k) + ... + a_{na}^i y(k - n_a) + b_1^i u(k) + ... + b_{nb}^i u(k - n_b) \quad (4.4)$$

then the nonlinear network is given

$$y(k+1) = A_0 + A_1 y(k) + ... + A_{na} y(k - n_a) + B_1 u(k) + ... + B_{nb} u(k - n_b) \quad (4.5)$$

which is an ARX model with its parameters defined at each operating point:

$$A_1 = \sum_{j=1}^{M} \rho_j(\psi) a_1^i, B_1 = \sum_{j=1}^{M} \rho_j(\psi) b_1^i, ... \quad (4.6)$$

This technique allows the use of a linear predictive controller and thus avoids the problems associated with computation time and optimality of the nonlinear solution. This technique can be viewed as a successive linearization around the operating point which yields linear MPC. The local model structure enables easy linearization of the nonlinear model due to its structure. For nonlinear systems the parameters of the linearized model can vary from the parameters of the system along the prediction horizon. The techniques to compute the model parameters along the prediction horizon are described in Chapters 4.4 and 4.5.

4.4 Single-model predictions

The predictor of the linearized model for the prediction horizon H_p is given by

$$\boldsymbol{y}(k) = \boldsymbol{P}\boldsymbol{x}(k) + \boldsymbol{R} + \boldsymbol{Q}\boldsymbol{u}(k) \quad (4.7)$$

where $\boldsymbol{P}\boldsymbol{x}(k) + \boldsymbol{R}$ is *free* response of the system that depends only on the past inputs and outputs and $\boldsymbol{Q}\boldsymbol{u}(k)$ represents *forced* response that depends only on future inputs.

$$\boldsymbol{y}(k) = \left[\, \hat{y}(k+1), ..., \hat{y}(k+H_p) \right]^T \tag{4.8}$$

$$\boldsymbol{x}(k) = \left[y(k), ..., y(k-n_a+1), u(k-1), ..., u(k-n_b+1) \right]^T \tag{4.9}$$

$$\boldsymbol{u}(k) = \left[u(k), ..., u(k+H_c-1) \right] \tag{4.10}$$

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & \cdots & p_{1(na+nb-1)} \\ \vdots & & \vdots \\ p_{H_p 1} & \cdots & p_{H_p (na+nb-1)} \end{bmatrix} \tag{4.11}$$

$$\boldsymbol{Q} = \begin{bmatrix} q_{11} & 0 & ... & 0 \\ q_{21} & q_{22} & ... & 0 \\ \vdots & \vdots & ... & \vdots \\ q_{H_p 1} & q_{H_p 2} & ... & q_{H_p H_c} \end{bmatrix} \tag{4.12}$$

$$\boldsymbol{R} = [r_1 .... r_{H_p}]^T \tag{4.13}$$

The coefficient of matrices ***P***, ***Q***, ***R*** for the affine second-order ARX models and prediction horizon equal to control horizon are given

$$\begin{aligned} &p_{11} = -a_1, && p_{12} = -a_2, && p_{13} = b_2 \\ &p_{21} = -a_1 p_{11} - a_2, && p_{22} = -a_1 p_{12}, && p_{23} = -a_1 p_{13} \\ &p_{i1} = -a_1 p_{(i-1)1} - a_2 p_{(i-2)1}, && p_{i2} = -a_1 p_{(i-1)2} - a_2 p_{(i-2)2}, \\ &p_{i3} = -a_1 p_{(i-1)3} - a_2 p_{(i-2)3} && \text{for } i = 3, ..., H_p \end{aligned} \tag{4.14}$$

$$\begin{aligned} &q_{ii} = b_1 && \text{for } i = 1, ..., H_c \\ &q_{(i+1)i} = b_2 && \text{for } i = 1, ..., H_c - 1 \\ &q_{ij} = -a_1 q_{(i-1)j} - a_2 q_{(i-2)j} && \text{for } j = 1..H_c \text{ and for } i = j+2...H_c \end{aligned} \tag{4.15}$$

$$\begin{aligned} r_1 &= a_0 \\ r_2 &= -a_1 a_0 + a_0 \\ r_i &= -a_1 r_{(i-1)} - a_2 r_{(i-2)} \end{aligned} \tag{4.16}$$

To eliminate steady-state errors caused by the modelling mismatch between the plant and model the estimated modelling error *err(k)* is computed at each sampling step

$$err(k) = y(k) - \hat{y}(k) \tag{4.17}$$

where $\hat{y}(k)$ is the 1-step ahead prediction at time (k-1). The estimate of error is then filtered to minimize the instability introduced by modelling error feedback. More complicated observers/compensators can be implemented within the control loop to prevent from the effect of model/plant mismatch that are responsible for static offset [58].

The predictive control algorithm consists of applying a control sequence that minimizes a cost function of the form

$$J = \sum_{j=1}^{N_p} \left(y(k+j \mid k) - w(k+j) \right)^2 + \lambda \sum_{j=1}^{N_c} \Delta u(k+j-1)^2 \qquad (4.18)$$

Expression (4.18) can be rewritten as

$$J = (\boldsymbol{Px} + \boldsymbol{Qu} + \boldsymbol{R} + \boldsymbol{T}err - \boldsymbol{w})^T (\boldsymbol{Px} + \boldsymbol{Qu} + \boldsymbol{R} + \boldsymbol{T}err - \boldsymbol{w}) + \lambda \boldsymbol{u}^T \boldsymbol{u} \quad (4.19)$$

where matrix $\boldsymbol{T}$ is defined as

$$\begin{aligned} t_1 &= 1 \\ t_2 &= -a_1 + 1 \\ t_i &= -a_1 t_{i-1} - a_2 t_{i-2} \end{aligned} \qquad (4.20)$$

The minimum of *J*, assuming there are no constraints on the control signals and $\lambda = 0$, can be found by making the gradient *J* equal to zero, which leads to:

$$\boldsymbol{u}(k) = [\boldsymbol{Q}^T \boldsymbol{Q}]^{-1} \boldsymbol{Q}^T [\boldsymbol{W}(k) - \boldsymbol{PX}(k) - \boldsymbol{R} - \boldsymbol{T}err(k)] \qquad (4.21)$$

where $\boldsymbol{W}(k) = [w(k+1), ..., w(k+H_p)]^T$ is a vector of future set-points and *err(k)* is an estimate of modelling error $e(k) = y(k) - \hat{y}(k)$.

Equation (4.19) with omitting the parts that do not involve the future control signal *u* and only shift the value of the cost function *J* can be rewritten as

$$J = \frac{1}{2} \boldsymbol{u}^T \boldsymbol{H} \boldsymbol{u} + \boldsymbol{b}^T \boldsymbol{u} \qquad (4.22)$$

where

$$\begin{aligned} \boldsymbol{H} &= \boldsymbol{Q}^T \boldsymbol{Q} + \boldsymbol{M}^T \boldsymbol{M} \\ \boldsymbol{b} &= \boldsymbol{Q}^T (\boldsymbol{Px} - \boldsymbol{w} + \boldsymbol{R} + \boldsymbol{T}err) + (\boldsymbol{N}^T \boldsymbol{M})^T \end{aligned} \qquad (4.23)$$

where the matrix M is defined as

$$\boldsymbol{M} = \begin{bmatrix} \lambda & 0 & \cdots & 0 & 0 \\ -\lambda & \lambda & 0 & \ddots & 0 \\ 0 & -\lambda & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \lambda & 0 \\ 0 & \cdots & 0 & -\lambda & \lambda \end{bmatrix}, \boldsymbol{N} = \begin{bmatrix} -\lambda \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(k-1) \qquad (4.24)$$

If constraints are introduced the quadratic programming (QP) problem has to be solved in every instant.

Implementation

1. Initialization - Construction of the LMN network based on *a priori* knowledge or process data using the training algorithms mentioned in Chapter 3.
2. Linearization – Based on model input vector, the LMN produces the one-step ahead prediction, which is used for calculation of the parameters of the linear model.
3. Control – Using the parameters of the linear system, the optimal output sequence is computed. Even though the control sequence is computed for the whole prediction horizon, only the first one is actually implemented.

The steps 2 a 3 are repeated every sampling period.

4.5 Multi-model predictions

In the single-model method, the local linear model at the current time k, $M(k) = (A_k, B_k)$, is used for the entire prediction horizon. For long prediction horizon, the influence of the approximation errors may significantly deteriorate the performance. This can be overcome by computing the model parameters along the simulated trajectory, i.e. a sequence of models *M(i)* is obtained for each sample in the prediction horizon. First scheme uses linearization about the predicted trajectory

system for the entire prediction horizon and the sequence pf models *M(i)* is obtained at one step.

In this case the prediction for a second order system

$$y(k+1) = -a_1{}^k y(k) - a_2{}^k y(k-1) + b^k{}_1 u(k) + b^k{}_2 u(k-1) \qquad (4.25)$$

with prediction horizon $H_p=2$ and control horizon $H_c=2$ is defined as:

$$\boldsymbol{y} = \begin{bmatrix} -a_1{}^k & -a_2{}^k & b_2^k \\ -a_1{}^{k+1} - a_1{}^k - a_2{}^{k+1} & -a_1{}^{k+1} - a_2{}^k & -a_1^{k+1} b_2^k \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} b_1^k & 0 \\ -a_1{}^{k+1} b_1^k + b_2 & b_1{}^{k+1} \end{bmatrix} \boldsymbol{u} \qquad (4.26)$$

where a_1^{k+1} represents the parameters of the model obtained from the linearization of LMN using the predicted output *y(k+1)* as a scheduling variable.

1. Linear model *M(k)* is used to compute the control signal *u* over the prediction horizon.
2. Simulate the system over the prediction horizon
3. Compute the parameters of the model at each point in the predicted trajectory to obtain $M(k+1).....M(k+H_p)$.
4. Use all the models $M(k+1).....M(k+H_p)$ to compute the control signal for entire prediction horizon.

Steps 3 and 4 are repeated until *u* converges. In order to provide convergence the method needs initial value for control signal. Single-model method as in steps 1 and 2 can be used to provide such an initial value.

In [60] different scheme has been proposed

1. Use the already obtained linear model *M(k)* and compute the control signal *u(k)* over the entire prediction horizon
2. Use *u(k)* to compute $y_m(k+1)$
3. Compute the parameters of the local model around the point $(y_m(k+1), u(k))$ to obtain *M(k+1)*

4. Use *M(k)* and *M(k+1)* and compute the new control sequence *u* for the whole prediction horizon
5. Take *u(k)* and *u(k+1)* and compute *y(m+2)*

Steps 3 through 5 are repeated for $i = k+1,....,k+H_p$. All the models $M(k).....M(k+H_p)$ are used to compute final control *u*. The above method is apparently slow due to its iterative character for control signal computation.

4.6 Internal Model Control based on Local Linear Models

When the model is available, the Internal Model Control (IMC) is one of the widely used approaches for control of the linear systems. The IMC scheme, first proposed in [61], has found a number of successful applications. Figure 13 shows the standard IMC control structure where G_s represents the transfer function of the process, G_M is the process model and G_R is the asymptotically stable transfer function. The feedback signal is the difference $y - \tilde{y}$ and the controller contains a model of the process explicitly. In fact, the IMC is a generalization of the Smith predictor.

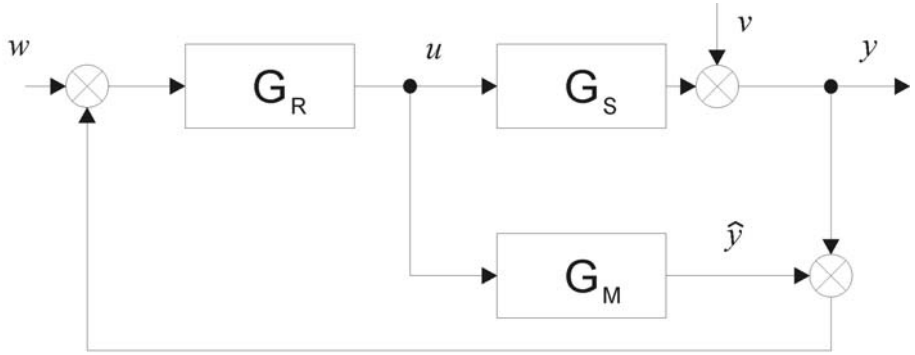


Figure 13 IMC block diagram

The IMC synthesis is a two step method. The Controller G_R is divided into two parts:

$$G_R = G_Q G_F \tag{4.27}$$

Firstly, the parameters of the controller G_R have to be determined. The best policy is to choose G_Q as an approximated inverse of the process model G_M, which will yield good tracking and disturbance rejection. In the second step the low-pass filter G_F is augmented to ensure robustness. The structure and parameters of G_F are chosen to achieve balance between robust stability and performance. The filter constant is the only parameter that has to be tuned, thus making the controller design simple. The IMC structure can be easily converted to the classical feedback control loop. If the process is of the first order time-lag system the controller will become PI [61].

The IMC approach can be easily extended to nonlinear models based on local model description [65]. In general, the inversion of nonlinear model is not simple and analytical solution may not exist. Let us consider the output of the local model network given by the equation

$$\hat{y} = \sum_{i=i}^{M} f_i(\boldsymbol{\varphi})\rho(\phi) \tag{4.28}$$

If the models f_i are defined as linear ARX type, the output of the local model network can be written as a nonlinear ARX model, i.e. NARX model. The parameters a_i, b_i of NARX model thus depend on the operating point. For the given NARX model the following exact inverse control law can be postulated:

$$u(k) = \frac{1}{b_1}\begin{pmatrix} v(k) - a_1 y(k) - a_2 y(k-1) - ..... - a_{na}(k - na - 1) - \\ -b_2 u(k-1) - .... - b_{nb} u(k - nb - 1) \end{pmatrix} \tag{4.29}$$

In IMC structure the *v(k)* replaces the y(k+1) which is not available at the time *k* by the filtered set-point response

$$y(k+1) \approx v(k) = G_F(w(k) - d(k)) \tag{4.30}$$

where $d(k) = y(k) - \hat{y}(k)$. In the case where IMC filter is chosen to be of the first order only one design parameter *c* is needed.

$$G_F = \frac{1-c}{1-cz^{-1}} \tag{4.31}$$

It has been proved in [61] that if the open-loop plant is stable, and if the model is perfect and inverse stable, the closed-loop system is also stable if the controller is the exact inverse of the model.

4.7 Concluding Remarks

There are two ways of designing controllers for local model structures, the linearization based and the local controllers approach. For linearization-based approach the local linear model is obtained from the nonlinear description through linearization and a single controller is designed based on the linear model. In the second approach a local controller is designed for each local model and the control output is then calculated as an interpolation of the local controller according to the actual operating point. There are two different possibilities how to manage the states of the local controllers. Either all the controllers may share the states or each controller has its own states.

Predictive control scheme that incorporates affine ARX model obtained through the linearization of the local model network has been proposed. In order to minimize the steady-state error arising in the standard MPC schemes due to model/plant mismatch and the model degradation through linearization the modelling error is included in the cost criterion for control sequence computation.

It may happen that at time sample k the control sequence is computed for model M_i but after applying control sequence $u(k)$ the plant is at operating point which is different from the model M_i. This can lead to bad performance if the difference between the models is large. To avoid such problems predictive control that uses linearization of the nonlinear model along the predicted trajectory can be implemented. The main advantage of this scheme is that it results in convex optimization problem.

Chapter 5 SIMULATION AND EXPERIMENTAL STUDIES

Several identification and control schemes that use multiple model description have been described in previous chapters. In Chapter 5 these techniques are tested on simulation experiments and laboratory models. First part of Chapter 5 presents the result on the simulation experiment of the pH neutralization plant. The pH neutralization plant represents a highly nonlinear process with dynamics dependent on the operating point which makes it suitable for testing nonlinear modelling schemes and control algorithms. The results of laboratory experiments on real plants are presented in the second part of the chapter. The chapter closes with discussion of results and comparisons.

5.1 Simulation Studies

This section is divided into two parts. In the first sub-section the identification of the structure and parameters of the local model network is studied. The second sub-section deals with the control of the nonlinear plant using the identified local model network.

5.2 pH Neutralization Plant

The studied system is a pH neutralization tank. A schematic diagram of the pH neutralization process is depicted in Figure 15. The neutralization process represents a

highly nonlinear process. The dynamic model used in this work has been developed in [62], [64] and has been used to test single loop strategies in [63].

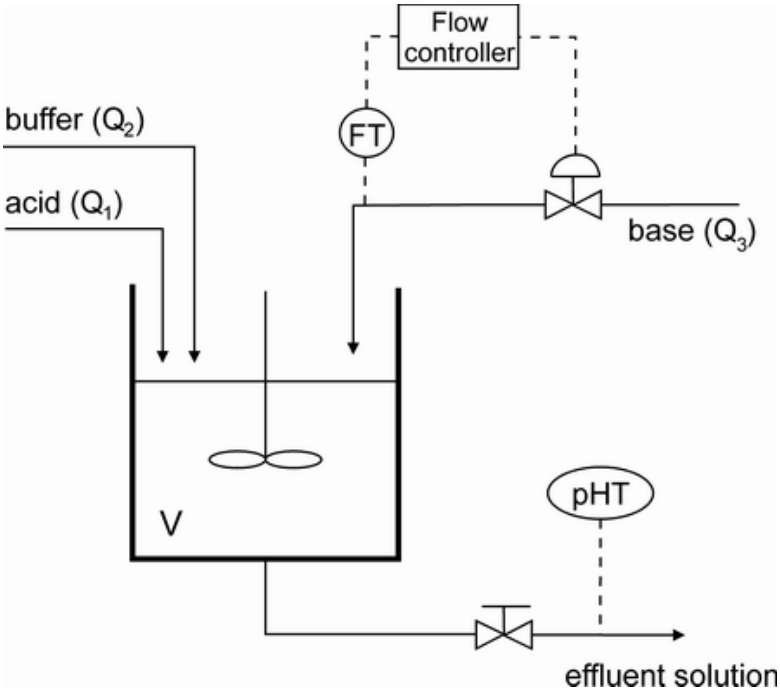


Figure 14 pH neutralization plant scheme

The process consists of an acid (HNO_3) stream, a buffer ($NaHCO_3$) stream and a base (NaOH) stream being continually mixed in the tank. The model is based on assumptions that the streams are perfectly mixed, the density is constant in the whole tank. The process aims at controlling the pH value (controlled variable) of the outlet stream by varying the inlet base stream Q_3 (control variable). The outlet flow-rate is dependent on the fluid height in the tank as well as the position of the valve.

The chemical reactions for the system are as follows:

$$\begin{aligned} H_2CO_3 &\longleftrightarrow HCO_3^- + H^+ \\ HCO_3^- &\longleftrightarrow CO_3^{2-} + H^+ \\ H_2O &\longleftrightarrow OH^- + H^+ \end{aligned} \tag{5.1}$$

The corresponding equilibrium constant are defined

$$K_{a1} = \frac{\left[HCO_3^-\right]\left[H^+\right]}{\left[H_2CO_3\right]} \tag{5.2}$$

$$K_{a2} = \frac{\left[CO_3^{2-}\right]\left[H^+\right]}{\left[HCO_3^-\right]} \tag{5.3}$$

$$K_w = \frac{\left[OH^-\right]\left[H^+\right]}{\left[H_2O\right]} \tag{5.4}$$

Reaction invariants are used to derive the pH process [62]

$$\begin{aligned} W_a &= \left[H^+\right] - \left[OH^-\right] - \left[HCO_3^-\right] - 2\left[CO_3^{2-}\right] \\ W_b &= \left[H_2CO_3\right] + \left[HCO_3^-\right] + \left[CO_3^{2-}\right] \end{aligned} \tag{5.5}$$

The invariant W_a is a charge related quantity, while W_b represents the concentration of the CO_3^{2-} ion. The pH can be determined from W_a and W_b, using an implicit equation:

$$W_a = \left[H^+\right] - \frac{K_w}{\left[H^+\right]} - W_b \frac{\frac{K_{a1}}{\left[H^+\right]} + \frac{2K_{a1}K_{a2}}{\left[H^+\right]^2}}{1 + \frac{K_{a1}}{\left[H^+\right]} + \frac{K_{a1}K_{a2}}{\left[H^+\right]^2}} \tag{5.6}$$

Solving the equation for $\left[H^+\right]$, the pH can be computed:

$$pH = \log_{10}\left[H^+\right] \tag{5.7}$$

A differential equation that describes the total mass balance of the tank is

$$\frac{dh}{dt} = \frac{1}{A}\left(Q_1 + Q_2 + Q_3 - c\sqrt{h}\right) \tag{5.8}$$

where c is a valve constant, A is the tank cross-section area and h is a tank level.

Differential equations for the effluent reaction invariants W_a and W_b can be derived:

$$\frac{dW_a}{dt} = \frac{1}{Ah}\left(Q_1(W_{a1} - W_a) + Q_2(W_{a2} - W_a) + Q_3(W_{a3} - W_a)\right) \tag{5.9}$$

$$\frac{dW_b}{dt} = \frac{1}{Ah}\left(Q_1(W_{b1} - W_b) + Q_2(W_{b2} - W_b) + Q_3(W_{b3} - W_b)\right) \tag{5.10}$$

where W_{ai} and W_{bi} are chemical reaction invariants of the *i*-th stream. The variables are defined in Table 1.

Symbol	Variable	Nominal value
A	Tank area	$207\ cm^2$
h	Tank level	$14\ cm$
Q_1	Acid flow-rate	$16.6\ ml/s$
Q_2	Buffer flow-rate	$0.55\ ml/s$
Q_3	Base Flow-rate	$15.6\ ml/s$
c	Valve constant	$8\ ml/s\sqrt{cm}$
W_{a1}	$[HNO_3]_1$	$0.003\ mol$
W_{a2}	$-[NaHCO_3]_2$	$-0.03\ mol$
W_{a3}	$-[NaHCO_3]_3-[NaOH]_3$	$-0.00305\ mol$
W_{b1}	$[NaHCO_3]_1$	$0\ mol$
W_{b2}	$[NaHCO_3]_2$	$0.03\ mol$
W_{b3}	$[NaHCO_3]_3$	$0.00005\ mol$
pK_{a1}	$-\log_{10}K_{a1}$	6.35
pK_{a2}	$-\log_{10}K_{a2}$	10.33
pK_w	$-\log_{10}K_w$	14

Table 1 Parameters of the pH neutralization plant

The initial conditions have been set to $W_a = 0.000436\,mol$, $W_b = 0.000528\,mol$, $pH = 7$. The estimation of the titration curve which shows the nonlinearity of the system was obtain by using several step function for the base flow-rate and recording the steady-state outputs. The experimental studies have shown that not only the static gain but also the time constants and dumping of the system are dependant on pH value.

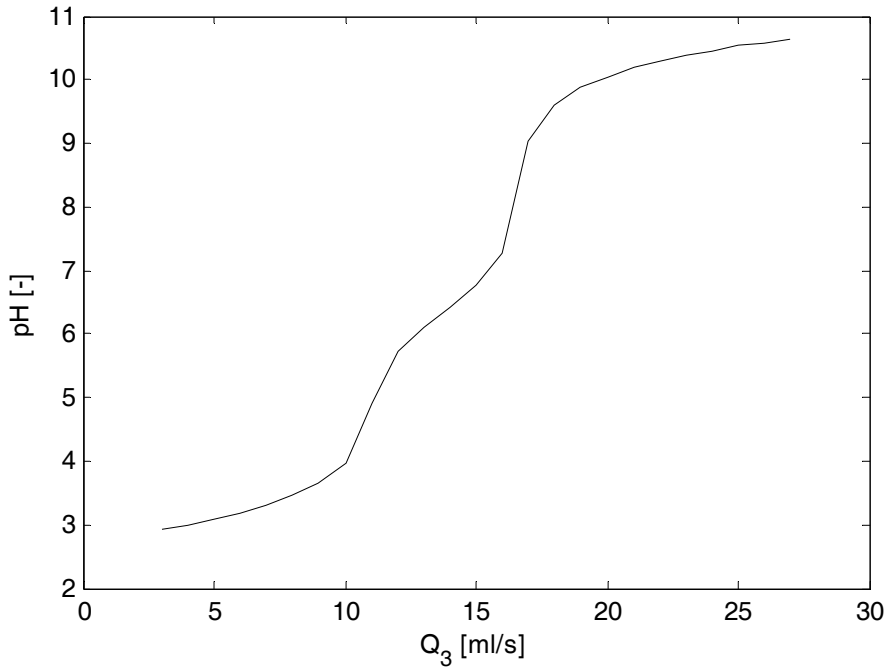


Figure 15 Titration curve

For the identification study, the acid and buffer flow-rates are kept constant at their nominal values and the base flow-rate Q_3 is used to excite the plant dynamics. From the titration curve it can be clearly seen that the process is nonlinear and thus suitable for evaluation and comparison of various local model identification and control algorithms.

5.3 Structure and Parameter Identification of the Local Model Network

This study uses data from the simulated pH neutralization plant. Training and test data were obtained using perturbations on the base flow-rate, with sample period of 15s. The local models were chosen to have the form of the second-order ARX model such that the local model network had the form:

$$pH(k) = \sum_{i=1}^{M} \rho\left(\boldsymbol{\psi}(k)\right) f_i(\boldsymbol{\varphi}(k)) \tag{5.11}$$

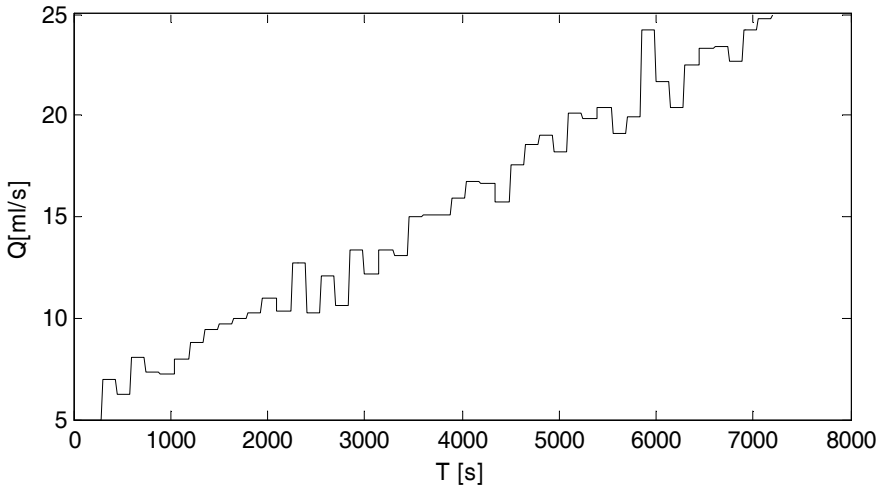
where $\boldsymbol{\psi}(k)$ is a vector of scheduling variables

$$\boldsymbol{\psi}(k) = pH(k-1) \tag{5.12}$$

The local models are given by

$$\begin{aligned} f_i(\boldsymbol{\varphi}(k)) = a_0^i + a_1^i pH(k-1) + a_2^i pH(k-2) + \\ + b_1^i Q_3(k-1) + b_2^i Q_3(k-2) \end{aligned} \tag{5.13}$$

The operating space of pH neutralization plant was restricted to $pH \in \langle 3,11 \rangle, Q_3 \in \langle 5\,ml/s, 25\,ml/s \rangle$. The training and test data were generated using perturbations on the flow-rate of the base stream. Training and test data used for training and validation of the local model network are shown in Figure 16 and Figure 17.



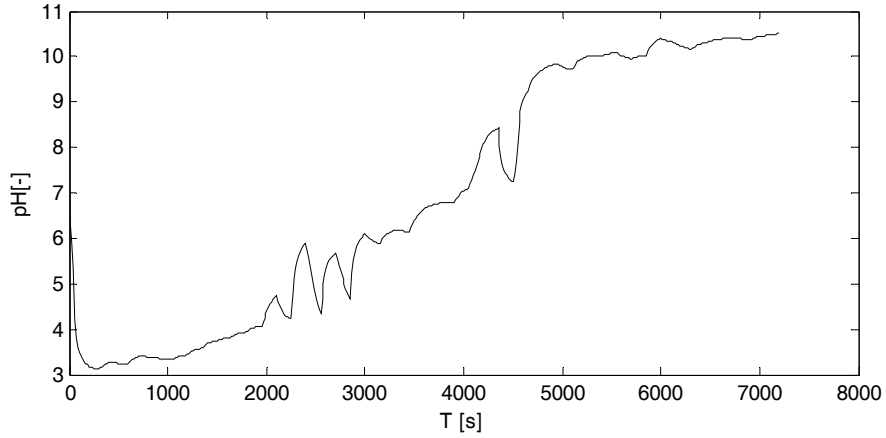


Figure 16 Training data for structure and parameter identification

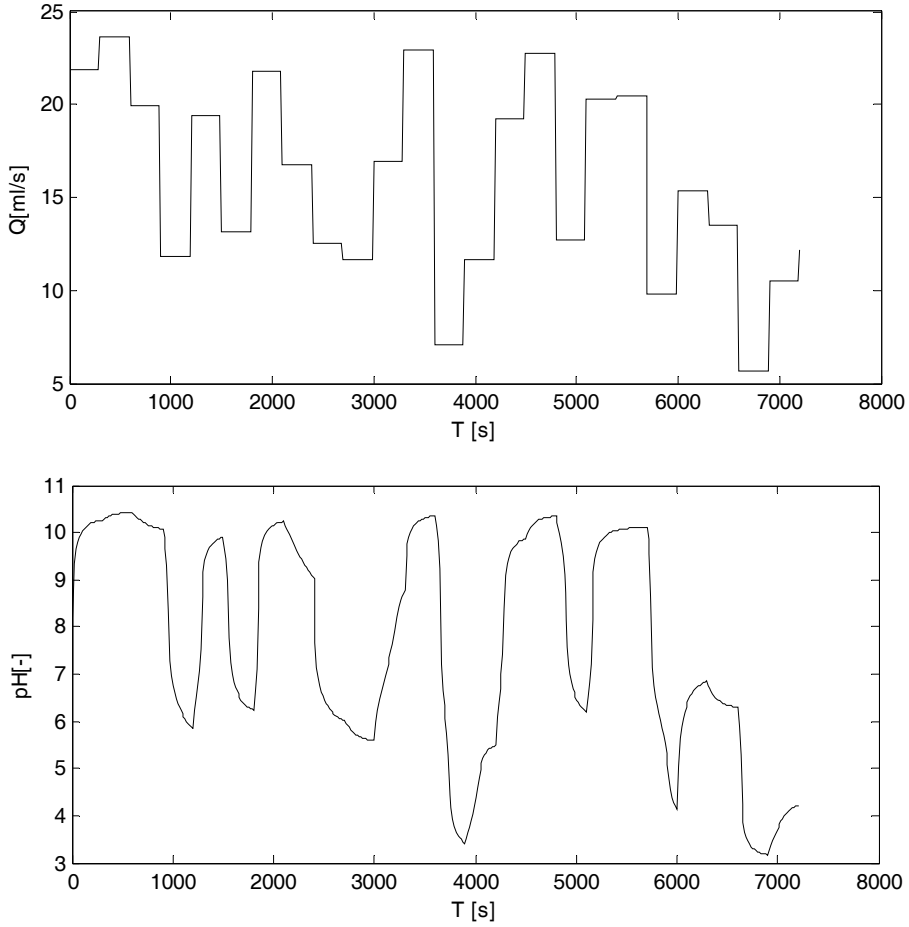


Figure 17 Test data for model validation

To compare the abilities of the identification algorithms several models were developed:

- a single ARX model
- a local model network with 5 equidistantly distributed models
- local model network whose parameters are optimized via Johansen and Foss algorithm
- LMN with all the parameters optimized using the evolutionary algorithm

In order to quantify the modelling performances of construction algorithms the mean sum squared error as in Equation (5.14) is used to measure the modelling performance. Here, *y* is a vector of measured outputs, $\hat{\boldsymbol{y}}$ is a vector of predicted outputs, i.e. infinite-step-ahead predictions. So the prediction $\hat{\boldsymbol{y}}$ at step *k+2* depends on the previous predictions $\hat{y}(k+1)$ and $\hat{y}(k)$, and model inputs $u(k+1)$ and $u(k)$.

$$J = \frac{1}{N}\left(\boldsymbol{y} - \hat{\boldsymbol{y}}\right)^T \left(\boldsymbol{y} - \hat{\boldsymbol{y}}\right) \tag{5.14}$$

Low value of the criterion signifies a good modelling performance. The static gain between the pH and the base flow rate varies considerably as the latter changes. From the titration curve five regions in which the gain is fairly constant can be recognized. So the growing optimization strategy was stopped when five local models were reached. The evolution algorithm used also five models to cover the whole operating space. Operating regions in Johansen and Foss algorithm were divided at 5 possible split points. The overlap parameter γ was set to 0.7.

For SOMA optimization, population of 20 individuals search for optimal solution for 50 migration steps.

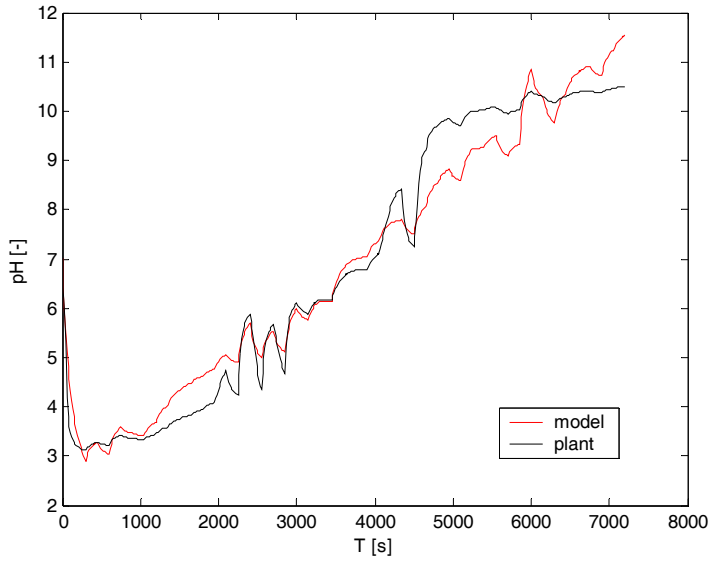


Figure 18 Parallel model predictions for training set – single model

In the following figures the parallel model predictions for training data set are shown. The black line shows the simulated nonlinear pH plant output and the red line represents the predicted output in each case.

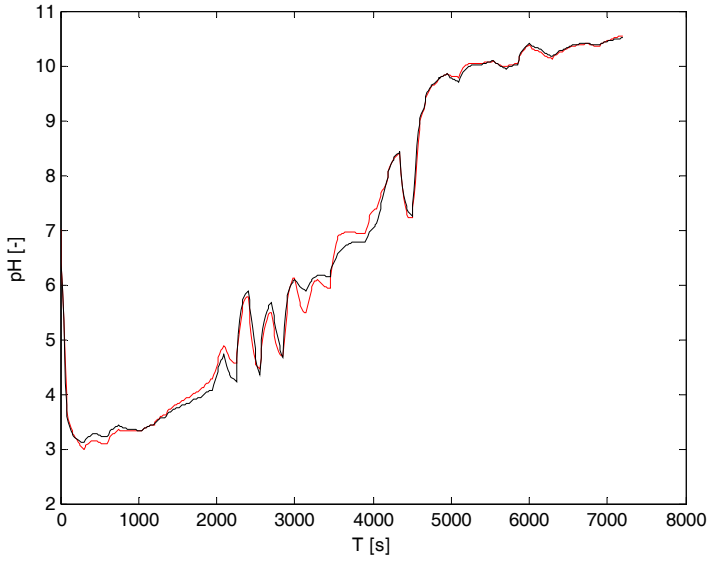


Figure 19 Parallel model predictions for training set – J&F algorithm with 5 models

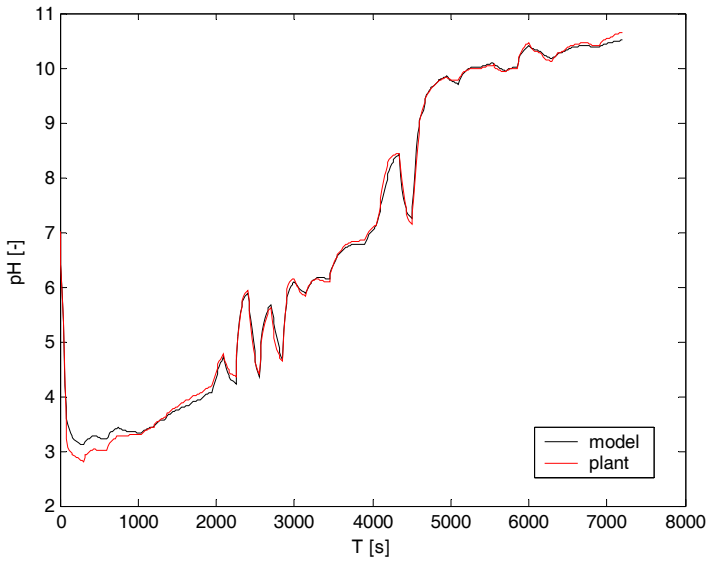


Figure 20 Parallel model predictions for training set – equidistantly distributed models

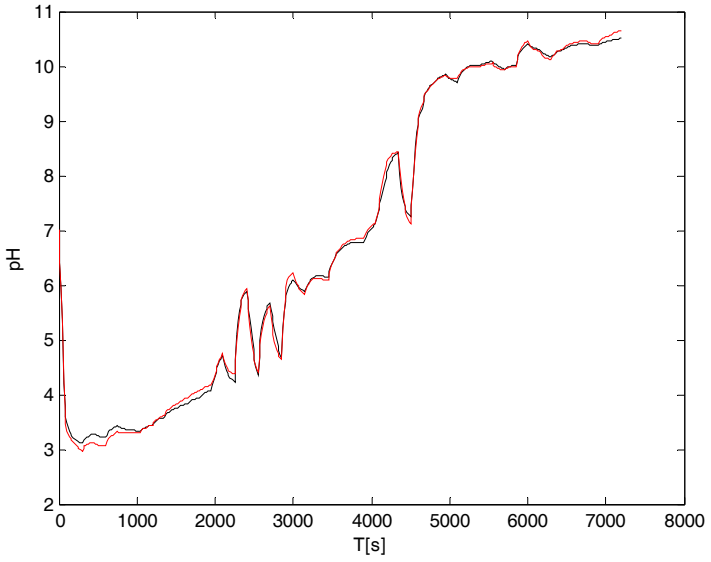


Figure 21 Parallel model predictions for training set – SOMA optimized

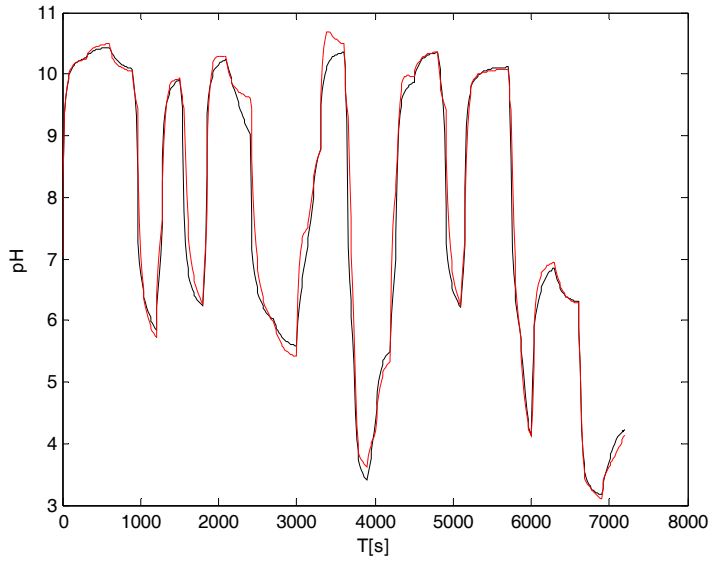


Figure 22 Parallel model predictions for test set – SOMA optimized

Network	***J* (Training Data)**	***J* (Test Data)**
Single ARX model	0.2759	0.3956
5 Equidistantly distributed Models	0.0117	0.1785
J&F	0.0174	0.2645
SOMA optimization	0.0082	0.1227

Table 2 Comparison of structure identification algorithms

Considerably better performance than the single model is obtained when using multiple models. Naturally, the *J* values for the test data are larger than for the training data in all cases. The values in Table 2 clearly demonstrate the modelling capabilities

of the 3 algorithm, with the SOMA evolution algorithm achieving the most accurate nonlinear representation, but for the highest computational costs. The validity functions parameters and transfer responses of local model parameters without the offset terms are shown in Figure 23 and Figure 24. These parameters were obtained with SOMA optimization of all the parameters. In all these figure one colour corresponds to one model.

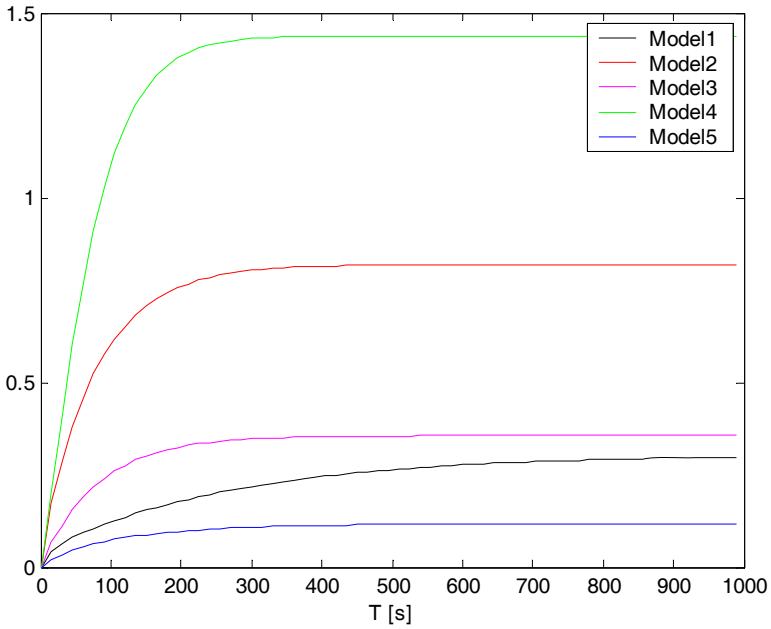


Figure 23 Responses of the local models for a step change of 1ml/s of the flow-rate in the corresponding operating point

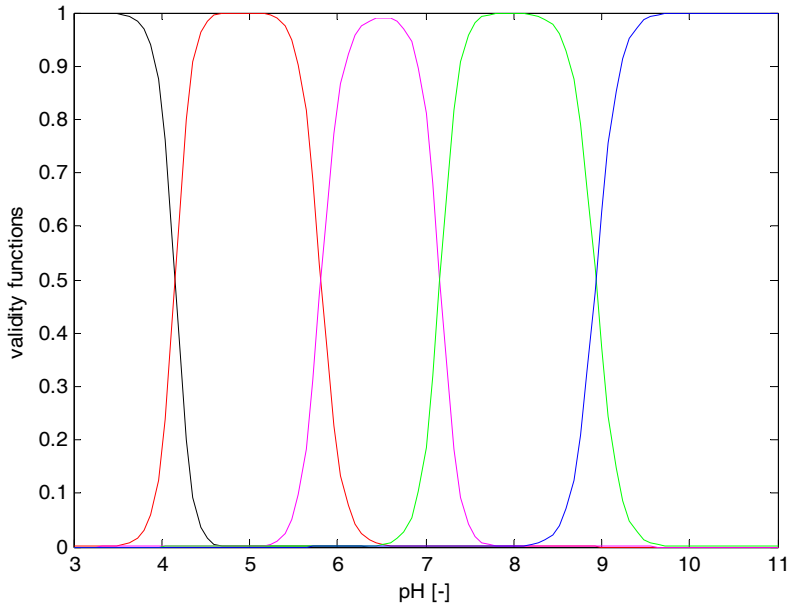


Figure 24 Validity functions of the local models

The bias term a_0 in the ARX local models provide that the models do not have a common intersection point in origin as can be seen in Figure 25.

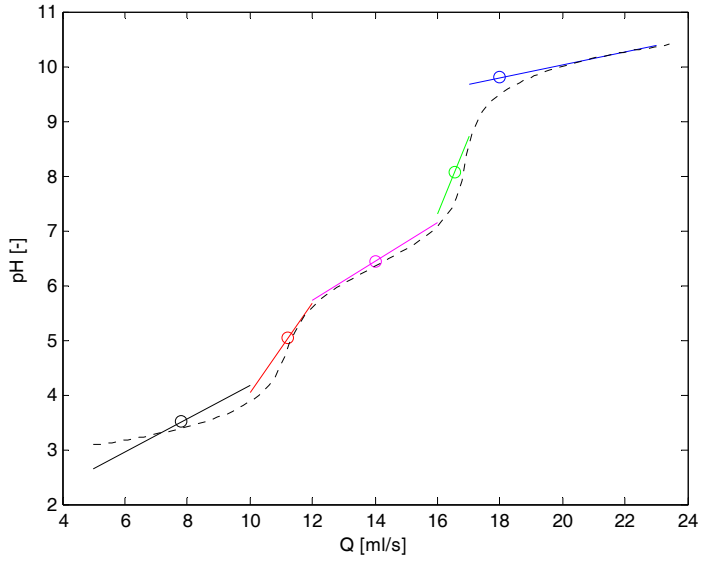


Figure 25 Local model network approximation of the nonlinear pH plant

5.4 Predictive Control using LMN

Like identification, the control of this nonlinear plant is hard due to strong dependency on the operating point. In this part of simulation studies the LMN-based predictive control is applied to control the pH value of the fluid flowing out of the tank. All the experiments are started with the same configurations of the LMN. The LMN used in these experiments was obtained using the SOMA optimization and its parameters can be found in the previous part. The LMN-based predictive control is compared with a predictive control that uses a single ARX model for predictions.

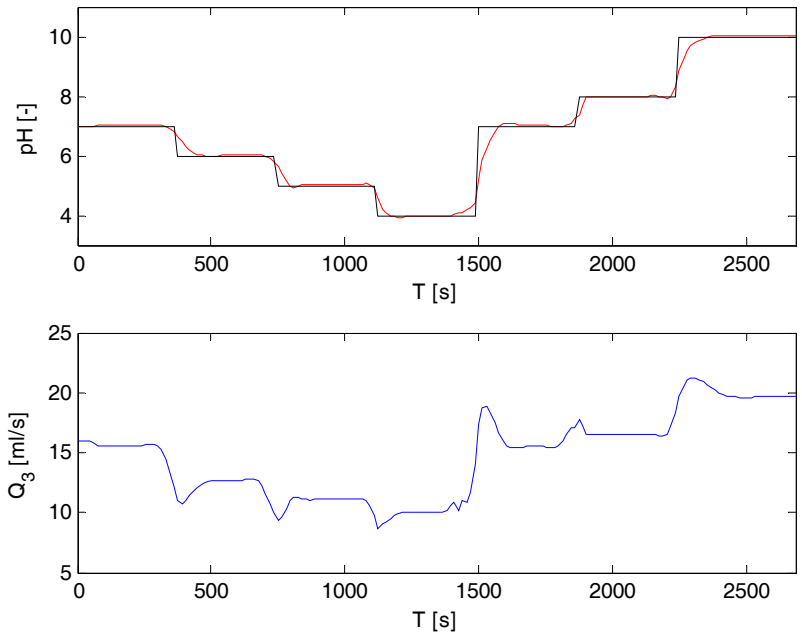


Figure 26 LMN-based predictive control (red-system output, black-reference signal, blue- control signal)

A control horizon and prediction horizon of 9 steps and the penalization constant 0.2 were for the design of predictive controller. The control results at Figure 26 and Figure 27 show that LMN-based predictive control technique performs better than a

single model and without steady-state error, which could be expected because a single ARX model cannot sufficiently represent nonlinear dynamics of the plant.

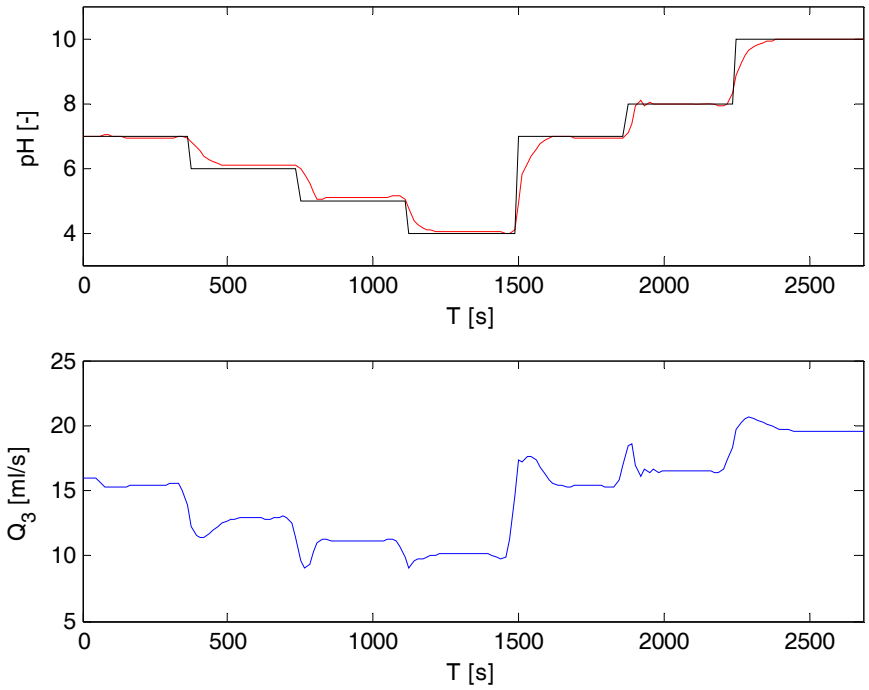


Figure 27 Predictive control with a single model (red-system output, black-reference signal, blue- control signal)

5.5 Nonlinear Model Predictive Control

For many-step-ahead control, the influence of the approximation errors may significantly deteriorate the performance of the predictive control. This can be remedied by computing the model parameters along the future trajectory. It can be seen that the controller based on the set of local models performs better than the controller that is using only a single local model along the whole prediction horizon. The improvement is larger if the set-point changes cross several local models as can be seen in Figure 28 between 1200 and 1600s when the plant outputs crosses all five operating regions with different dynamics. In the experiment the same model as in the previous examples is used.

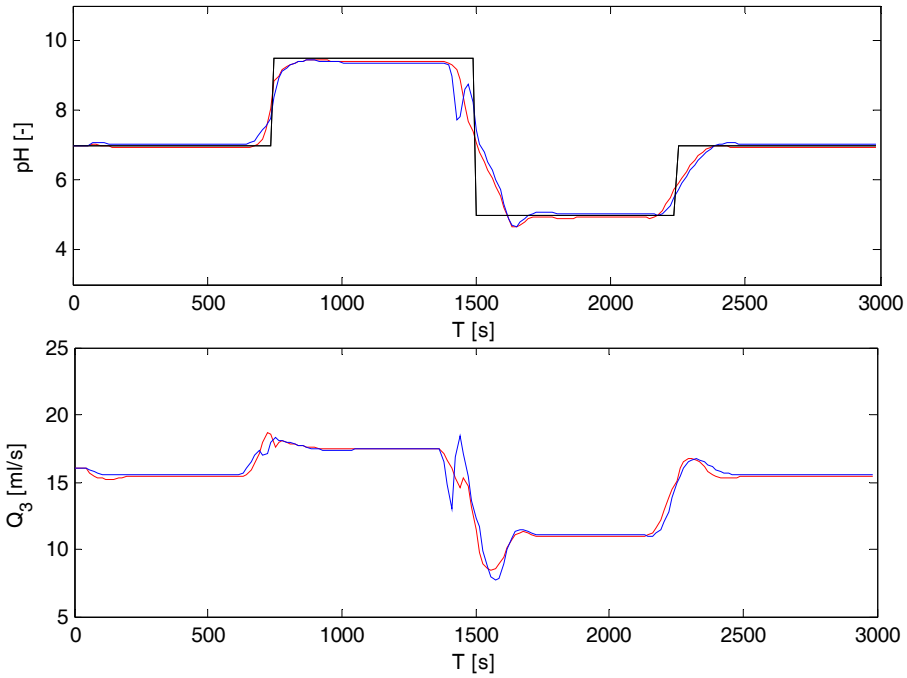


Figure 28 Comparison of single and multiple model predictions (blue-single model, red-multiple model)

5.6 Internal Model Control using LMN

The IMC control strategy was applied to control the pH in pH neutralization plant. The same LMN network scheduled on pH as in the predictive control scheme was used to model the plant dynamics. The robustness filter

$$G_F = \frac{0.02}{1-0.98z^{-1}} \tag{5.15}$$

was added to the analytical inverse of the model.

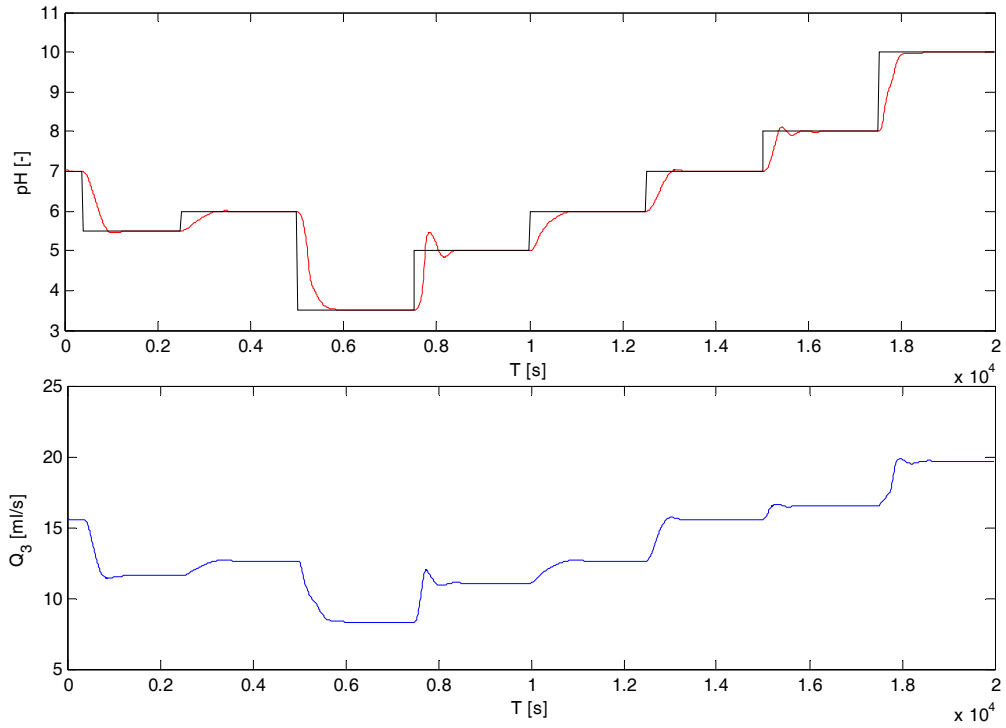


Figure 29 IMC control of pH neutralization plant – set-point tracking

Figure 30 compares the set-point tracking performance of the internal model controller that uses LMN as internal model with IMC controller that uses only a single linear model during the whole experiment.

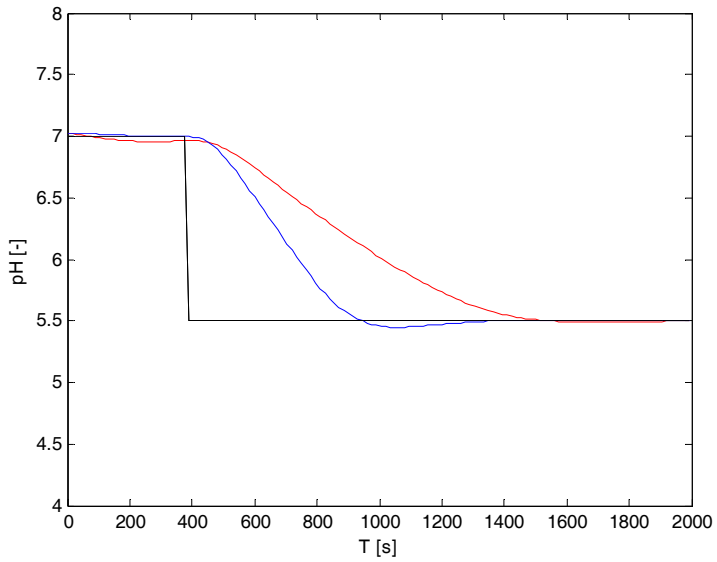


Figure 30 Setpoint tracking (blue –LMN based IMC, red-IMC with linear model)

Figure 31 shows the disturbance rejection capabilities of the IMC controller for the change of buffer flow rate from 0.55 to 0.15 ml/s at the time t = 200s. This reduction significantly increases the process gain.

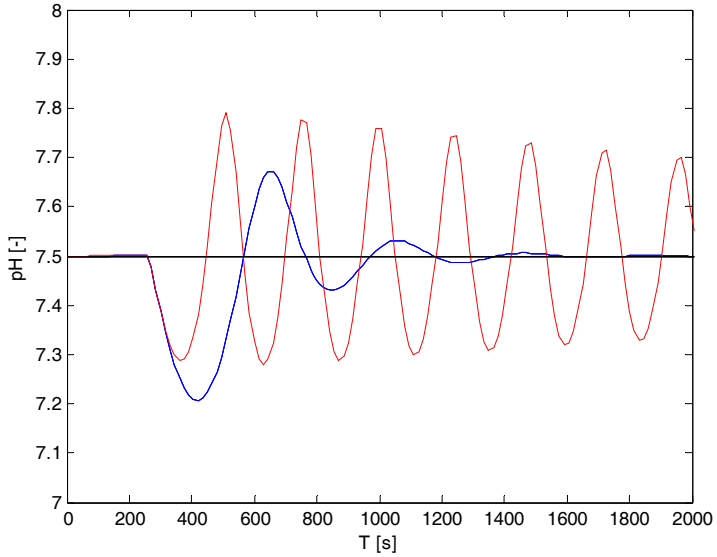


Figure 31 Disturbance rejection (blue –LMN based IMC, red – IMC with linear model)

5.7 Laboratory Experiments

This section deals with the application of proposed identification and control algorithms to real plants. The three-tank model has been chosen for this purpose because it demonstrates nonlinear behaviour. The laboratory process consists of three plexiglas cylindrical tanks with identical cross-sections supplied with distilled water. The liquid levels are measured via the piezoresistive transducers. The laboratory model is depicted in Figure 32. Water is supplied to the first tank through a controlled pump. Additionally, another connection to the reservoir lies at each tank, enabling the introduction of disturbances in the form of leaks. The process is connected to the PC through a data acquisition board. The identification and control software is written under MATLAB, using the Real–Time Toolbox to collect data from the process. The sampling time in all the experiments is set to 1s. The level of the liquid in the first tank is measured and its value is limited from 0 to 0.6m.

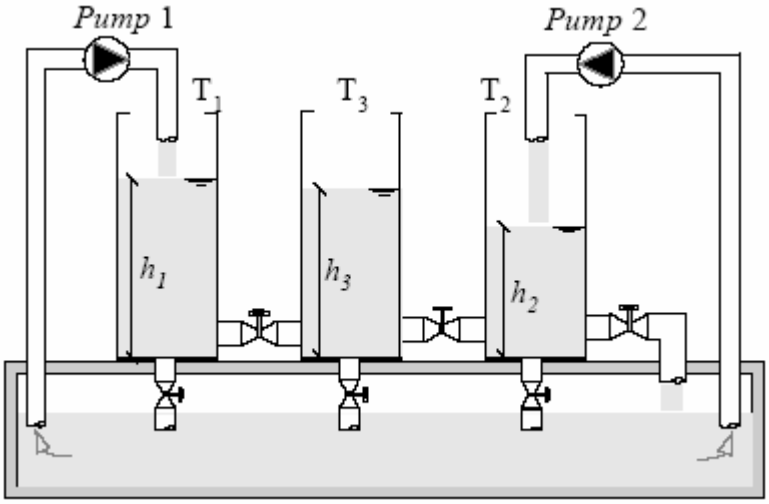


Figure 32 Scheme of the three-tank system

For all the experiments only the tanks h_1 and h_2 are used. The task is to control the level in the tank h_1 using the voltage applied to the *pump 1*. The valve between the tank T1 and T3 is fully open. The valve from the tank T1 is closed and the valve from tank T3 is partially open.

5.8 Nonlinear Modelling and Control of the Two-tank System

A LMN was used to model the nonlinear dynamic relationship between the input voltage to the pump and the tank level from the process operating data. Since the input signal in MATLAB units lies in range (-1, 1), signal was shifted by adding 1 to the input signal. Thus input signal can vary from 0 to 2, where 0 means that the pumps are off. The process operation is divided into three operating regions. Two data sets were collected from the process. During the experiments random signal was applied to the system, with the sampling period of 5 second. The structure of the network was chosen to be affine local ARX models of the second order.

$$y(k+1) = a_0 + a_1 y(k) + a_2 y(k-1) + b_1 u(k) + b_2 u(k-1) \qquad (5.16)$$

The centres of validity function were restricted to lie within the operating region (0 m, 0.1 m), (0.1 m, 0.3 m), (0.3 m, 0.6 m) respectively. The dynamics of the models is assumed to be stable and the gain of local plant must lie within the limits (0.01 m/MU, 3 m/MU). The previous limits on the process behaviour where translated to the form of inequalities. For the optimization of the validity function parameters the SOMA algorithm was used in combination with identification of the local model parameters using quadratic programming with constraints. Figure 33 shows the training and test data used for network training and validation. Comparing the step responses of the local models, one can find that the local model 1 has lower gain and time constant.

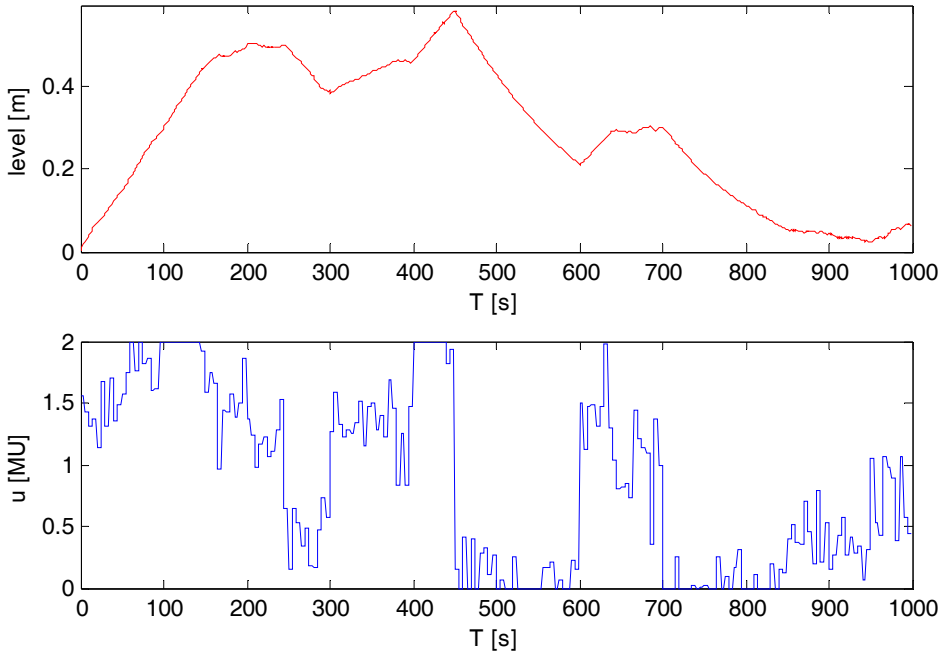


Figure 33 Data for training the LMN

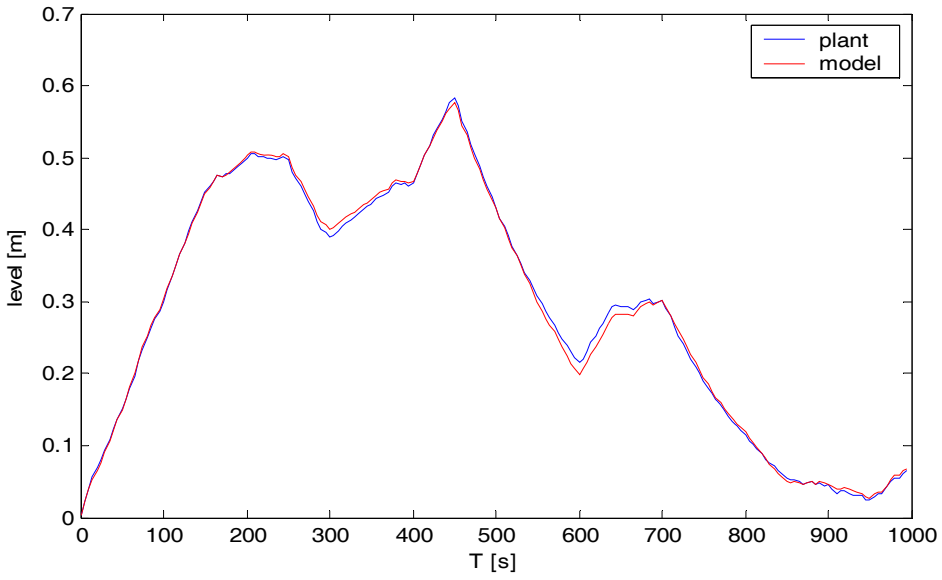


Figure 34 LMN prediction on training data

The LMN based predictive controller was developed to control the tank level. For control purposes the constrained optimal control problem is solved with a sampling period of 5 second and choosing the prediction and control horizons to be H_c =20 and H_p=20 time steps. The penalization constant $\lambda = 0.05$ was used within the criterion to stress the importance of the control error. Assuming the physical bounds on the flow-rate supplied by the pumps, the constraints (0, 2) were imposed into the optimisation problem. Figure 35 shows the performance of the LMN based predictive controller.

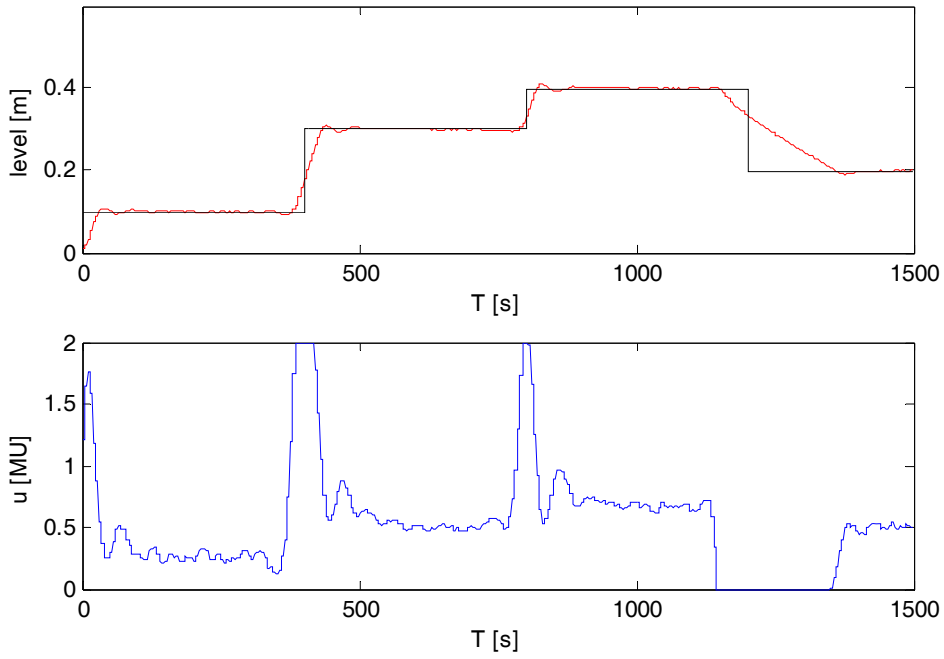


Figure 35 Predictive control with LMN network

To compare the capabilities of the standard linear MPC without any offset compensation was used in the same experiment. As can be observed the standard MPC guarantees a stable response without constraints violation, but with static offset. These deviations can be attributed to the fact that there is no offset compensation. The effect of the nonlinear model in the predictive controller is minor.

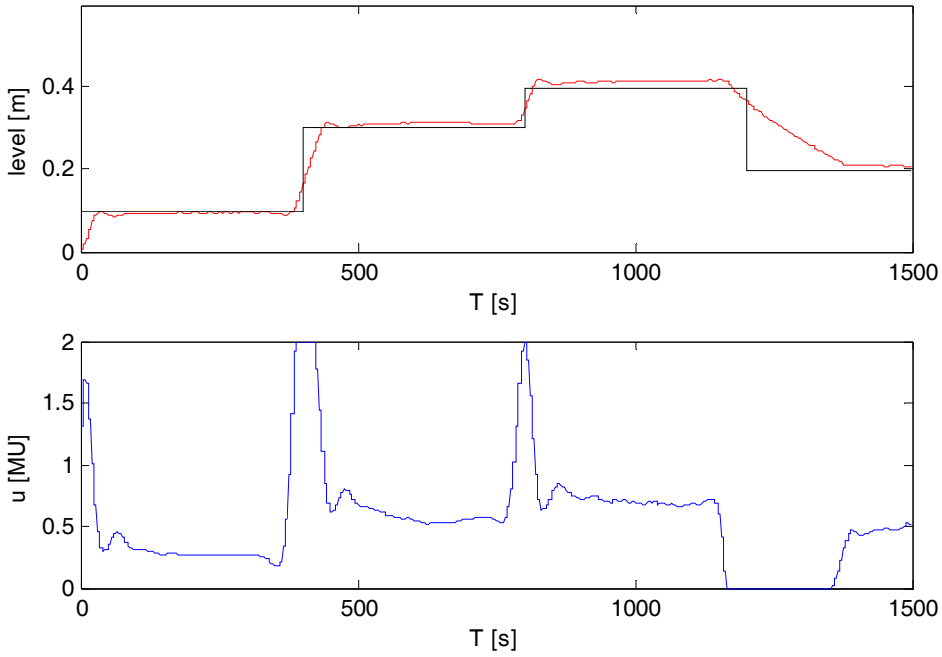


Figure 36 Single model predictive control

5.9 Internal Model Control of Two-tank system

The IMC control strategy was applied to the control of the level in the first tank. The parameters of the model for computation of the model inversion were obtained by scheduling on the level at the step *k-1*. Figure 37 demonstrates the performance of the LMN controller. The filter

$$G_F(z^{-1}) = \frac{0.1}{1-0.9z^{-1}} \tag{5.17}$$

was added as part of the controller to improve the robustness to model uncertainty.

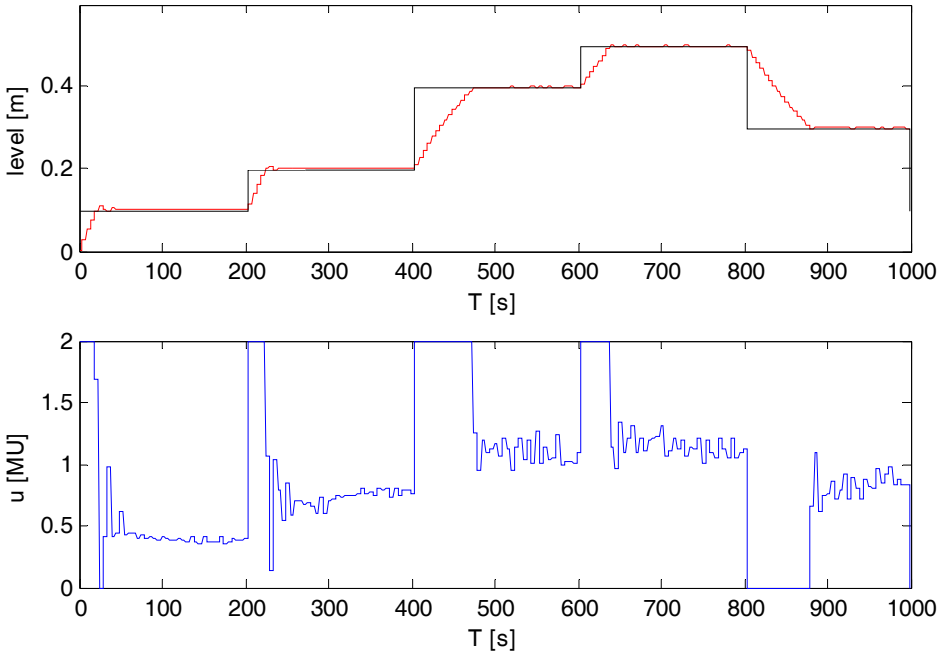


Figure 37 Internal Model Control of the level in the first tank

The IMC control enables easy application of LMN network, where only the parameters of the filter have to be specified. The control performance of the IMC based on LMN is also satisfactory.

5.10 Discussion and Concluding Remarks

By using a pH neutralization model plant it has been shown that LMN approach can provide an improvement in modelling accuracy over a single linear model when used to represent a dynamics of a chemical plant over a wide operating range. The modelling performance of J&F algorithm and SOMA optimization algorithm were evaluated using simulated data from pH neutralization plant. The SOMA optimization achieves an optimal model but at the expense of intensive computation, while J&F algorithm produces less accurate model also with local interpretability but with much less computational effort.

The incorporation of the prior knowledge to the parameter identification allows building models that accurately approximate the plant both globally and locally. Moreover, it allows modelling of accurate models where less experimental data is available.

The Model Predictive Control algorithm uses linearization of the LMN model at current operating point to compensate nonlinearity in process dynamics. The control algorithm was applied for control of the pH neutralization plant. In experiments better performance in terms of set-point tracking was achieved comparing to a single model representation of the process. In order to minimize the steady-state off-sets the internal loop is implemented to the control loop by changing the reference signals supplied to standard MPC structure. As could be observed the incorporation of the offset compensator contributes to remove the static offset caused by the model/plant mismatch.

A situation when at time k the controller output signal $u(k)$ drives the nonlinear plant to a operating region represented by another local linear model can occur. In this case, the control signal is computed for one system and is applied to control another system. This can badly affect the quality of control if there is a considerable difference between the dynamics of both models. This can be overcome by using the predicted trajectory to compute the future linearized models on the predicted trajectory. Higher value of penalization constant λ in the quadratic cost function J which prevents from fast changes of the controlled variable can also prevent from mentioned undesired effects.

LMN with ARX local model can be easily incorporated into the internal model control framework since the required model inverse can be analytically obtained.

Simulation results for a pH neutralization plant show the significantly improved set-point tracking and disturbance rejection of both control schemes compared to the single linear model. The experiments on the three-tank system show that the only small improvement can be achieved with LMN description of the process. This can be contributed to the fact that level control in the first tank that is of cylindrical shape is a weakly nonlinear process.

Chapter 6 Concluding Summary and Future Work

This thesis has examined the LMN structures for identification and control of nonlinear systems. The idea beyond these networks is to divide the operating range of nonlinear system into smaller part where the nonlinear system can be represented by a simple model. In combination with weighting function relatively accurate approximation can be achieved. LMN models are also easier to interpret than nonlinear models based on MLP networks and are suitable for building a nonlinear controller that is composed of local linear controllers. The LMN network can be viewed as generalization of Radial Basis Function (RBF) network and under specific condition is functionally equivalent to Takagi-Sugeno fuzzy model. Although normalization of the weighting function is necessary to provide partition of unity it can also cause several undesired side-effects such as reactivation or loss of local support.

The learning algorithms can be divided into two classes: local learning techniques, which are less computationally expensive and create models which are closer to local linearizations of the nonlinear dynamics and global learning techniques with better global fit. In Chapter 2 several training algorithms are described and new training algorithm that utilizes Self-Organizing Migration algorithm for validity function optimization and quadratic programming for local model parameters estimation is proposed. This method is able to find improved model compared to the heuristic Johansen and Foss algorithm. Allowing the splits to be at any point of the operating space can reduce the number of models and achieve better accuracy. On the other hand, large number of model evaluations is necessary for optimization. Also the number of models has to be specified beforehand while heuristic algorithms constantly increase the number of models and user can choose the best compromise between the accuracy of the model and its complexity.

One the main advantages of the LMN is easy integration of *a priori* knowledge. If prior knowledge about the system is available it can be included in the training algorithm in the form of inequalities. In this case the conventional least-squares method or weighted least-squares method is substituted by quadratic programming.

The construction algorithms J&F and SOMA have been compared and their modelling performances were illustrated using both simulated and real data. The obvious trade of between the computational complexity and the model quality has been highlighted. The SOMA optimization algorithm searches for an optimal model but at the expense of complex computation.

The predictive control scheme that uses linearization of LMN for prediction future outputs and compensation for model/plant mismatch has been proposed. Results from experiments have shown that incorporation of the modelling error into the cost criterion ensures good tracking performance despite modelling errors. If the LMN consists of ARX models, the required model inverse can be analytically computed and thus can be implemented in the Internal Model Control scheme. Simulation results show the improved performance of the proposed schemes compared with a single model.

Results of various simulation studies have been presented in Chapter 5 that showed the satisfactory performance of the proposed identification schemes. All the experiments and simulations within the thesis have been only on single-input single-output systems. Since the predictive control can be easily adopted for MIMO systems the future research would involve predictive control of MIMO systems. During the design of LMN-based control schemes, the global stability of the closed-loop system has not been investigated and may be a part of future research.

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Publications

Most of the work in this thesis has been presented in international and national conferences. The conferences are listed below:

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